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CONCRETE GEOMETRY

T. NELSON

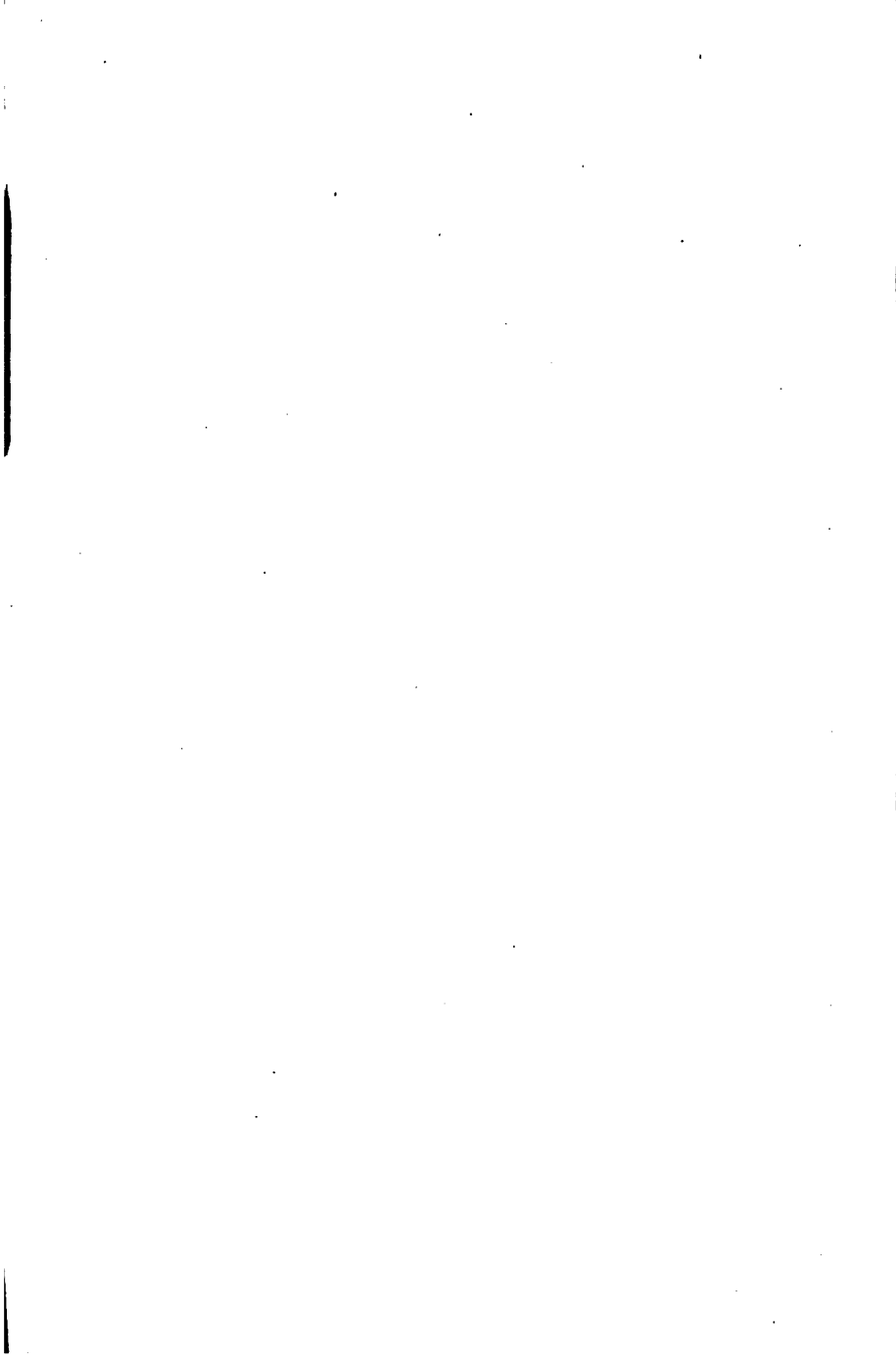
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CONCRETE GEOMETRY

INTRODUCTORY TO FORMAL PLANE GEOMETRY

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TO THE TEACHER.

This book has been prepared primarily for class-room work only, although it may be used equally well for outside work also. The best results will be obtained by having each pupil provided with a blank book with flexible cover so that it may be opened out flat after the manner of a copy book, the pages being of a size sufficient to accommodate a figure at the top and, below this, the answers to questions referring to the figure. The work in this blank book should be divided into sections numbered to correspond with those in the text-book. The answer to every question should be written in the form of a complete statement. By the use of such a book the work of each pupil is easily preserved, and back work can be referred to at any time. The blank book will also be found useful for the purpose of reviewing. It is obvious that accuracy and neatness are essential.

It is intended that the majority of the definitions and geometric principles be deduced by the pupils themselves. Spaces have been left for the purpose of writing these definitions and statements in their proper places in the body of the text as well as in the blank book; but they should not be written in the text-book until the class, as a whole, under the guidance of the teacher, has learned to express them in a satisfactory manner. The purpose of this book is to teach geometric facts and how to use them; but, although we have endeavored to make the deduction of these facts as obvious as possible, it is not intended, in the majority of cases, to hold the pupil responsible for the deduction. That belongs more properly to the study of Formal Geometry. At the end of the book will be found

model examination papers on each chapter. These are designed to suggest to the teacher, who may be more familiar with the requirements of Formal Geometry, how far and in what way he should expect the pupil to be familiar with the geometric properties of plane figures, with the abstract statement of those principles, and with the logical methods used occasionally for establishing the truth of a general statement.

It is evident that, in order to obtain satisfactory results, the teacher should have a thorough knowledge of Demonstrative Geometry, and be familiar with the wording of its definitions and theorems. He should repeatedly call the attention of students to the fact that the deductions in this book from particular concrete examples are not demonstrations; but that in common life we attach to such conclusions a high degree of probability.

Where it is undesirable to have the text-book marked by the pupil, the definitions and statements may be written in the blank book only, and marked with the proper section number to admit of easy reference.

LIST OF TOOLS.

In order to make the constructions and measurements required, each pupil should be provided with the following tools :

Ruler, twelve inches long.

Compasses, five and one half inches.

Scale, twelve inches long, with one edge divided into sixteenths of an inch and the other into centimeters and millimeters.

Protractor. The best form of protractor for this purpose is one of horn or celluloid, divided into degrees numbered both ways, and not more than four inches in diameter.

45° triangle, five-inch edge.

30°×60° triangle, five-inch edge.

Lead pencil.

Eraser.

Scissors.

The teacher should have a scale, one meter long, divided into decimeters and centimeters on one side and into inches and eighths of an inch on the other, two large triangles, and a large paper protractor for blackboard work and for making explanations to the class. It is also necessary to have two wooden cylinders, with diameters of 7 cm. and 1 dm., respectively. (See § 157.)

Tools may be ordered with the books, if desired.

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CHAPTER I.

STRAIGHT LINES AND ANGLES.

1. We are all more or less familiar with the meaning of the word *surface*. We know what is meant by the surface of the earth, the surface of a pond, the surface of a table, etc.

It is clear that a surface has no *thickness*, but is merely the outside or boundary of any solid object.

In *elementary geometry* we deal with only a few well defined surfaces, such as the surface of a sphere, called a *spherical surface*; the surface of a cylinder, called a *cylindrical surface*; the surface of a cone, called a *conical surface*, and the plane surface called a *plane*.

The surface of a ball or soap bubble is an illustration of a spherical surface.

The surface of a lead pencil or gas pipe is an illustration of a cylindrical surface.

The surface of a funnel is an illustration of a conical surface.

(The different surfaces should be illustrated to the class by models.)

We shall deal with only one of these surfaces, namely, the **plane**.

Of the many illustrations of a plane we may take the surface of a small pond, which is a plane surface when it lies perfectly smooth and undisturbed by wind or other force.

The intersections of two surfaces or the boundaries of a surface are **lines**.

The edges of a table are lines. The intersections of the walls of a room are lines.

(Other illustrations of lines may be shown by models.)

A line has *length* but no breath or thickness.

2. A sheet of paper laid flat on the table or desk may be taken to represent a portion of a plane, and lead pencil marks on this paper may be taken to represent lines in the plane.

Fig. 1 represents three kinds of lines.

AB is a **straight line**.

Definition:—A *straight line* is a line which has the same direction throughout its entire length.

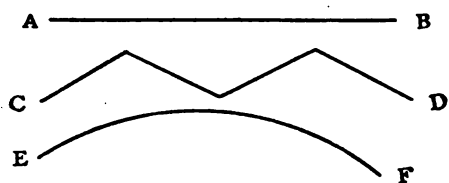


FIG. 1.

*Axiom*¹:—A straight line is the *shortest line* that can be drawn from one point to another.

CD is a **broken line**.

Definition:—A *broken line* is a line made up of different straight lines.

EF is a **curved line**.

Definition:—A *curved line* is a line no part of which is straight.

3. A straight line is named by placing letters at its extremities.

In Fig. 2 we have the three straight lines AB, CD, and EF.

The extremities of a line and the intersections of lines are **points**.

A point has no length, breadth, or thickness.

A point is named by a letter placed near it.

If we wish to represent a point not on a line we use a dot.

¹An *axiom* is a truth assumed as self evident.

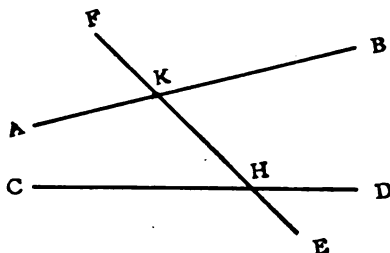


FIG. 2.

In Fig. 2 how many and what kind of lines are there?

How many points are lettered?

Name those which are extremities of lines.

Name those which are intersections of lines and tell

what lines intersect at those points.

4. Two straight lines may differ in *length* or in *direction* or in *both length and direction*.

The length of a line is determined by observing the number of times that a certain fixed length, called the *unit of length*, is contained in it.

The operation of determining the length of a line is called *measuring* the line.

A straight line is measured by means of a *scale*, which is a ruler with the edge divided into units of length.

5. In this book we shall use two systems of measure, the **English** and the **Metric**.

The English units of length are the **inch, foot, yard**, etc.

The following table gives the relations between these units:

12 inches (in.)	equal 1 foot. (ft.)
3 feet	" 1 yard. (yd.)
$5\frac{1}{2}$ yards	" 1 rod. (rd.)
320 rods	" 1 mile. (mi.)

The abbreviation for each unit is given in parenthesis.

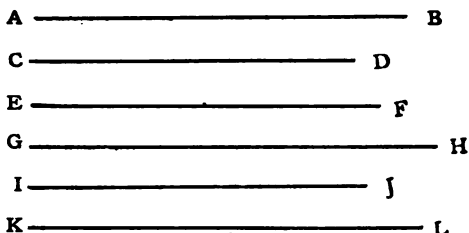
The metric units of length are the **meter** and other units derived from the meter by multiplying or dividing it successively by ten.

The following table gives the relations between these units of length :

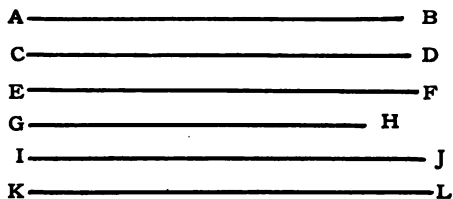
10 millimeters (mm.)	equal 1 centimeter.	(cm.)
10 centimeters	" 1 decimeter.	(dm.)
10 decimeters	" 1 meter ..	(m.)
10 meters	" 1 dekameter.	(dkm.)
10 dekameters	" 1 hektometer.	(hm.)
10 hektometers	" 1 kilometer.	(km.)

6.**EXERCISES.**

1. Measure the lines below, obtaining their lengths in inches and fractions of an inch.

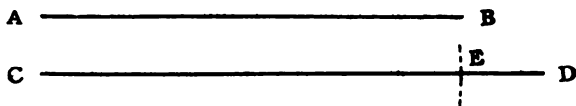


2. Measure the lines below, obtaining their lengths in Metric units.



7. A straight line is drawn by the aid of a ruler, the edge of which is a straight line.

Problem:—*To draw a straight line equal to a given straight line.*



Let AB be the given straight line.

With your ruler draw any straight line such as CD longer than AB.

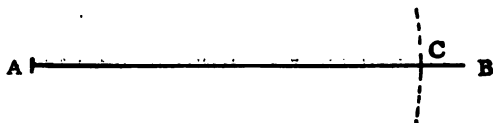
(This you can judge with the eye.)

Spread your compasses and place the needle point on A and the pencil point on B.

Without changing the spread of the compasses, place the needle point on C and with the pencil point mark the point E on the line CD.

Then CE is equal in length to AB.

8. Problem:—*To draw a straight line equal to a given length.*



Suppose that we wish to draw a line 2 inches long.

With the ruler draw any straight line AB longer than 2 inches.

(This can be judged by the eye.)

Using the scale, mark a point C on the line AB which shall be 2 inches from A.

Then AC is a line 2 inches in length.

9. EXERCISES.

1. Draw straight lines from left to right on your paper, and with the aid of your compasses make them equal to the lines given in exercise 1 of §6.

2. Draw straight lines up and down on your paper, and with the aid of your scale make them equal in length to the lengths obtained for the lines given in exercise 2 of §6.

3. If $x=2$, draw a straight line x inches in length.
4. If $y=1\frac{1}{2}$ draw a straight line y in. long.
5. Draw a straight line $x+y$ inches long taking the values of x and y given above.
6. If $x=5\text{cm. } 3\text{mm.}$ and $y=6\text{cm. } 7\text{mm.}$, draw a line whose length is $x+y$.
7. If $x=19\text{cm. } 8\text{mm.}$ and $y=10\text{cm. } 3\text{mm.}$, draw a line whose length is $x-y$.
8. If $x=2$, $y=6$ and $z=\frac{1}{2}$:
 - i. Draw a line $x+y+z$ inches long.
 - ii. Draw a line $x \times y \times z$ inches long.
 - iii. Draw a line $(y+z) \div x$ inches long.
 - iv. Draw a line $x+y-z$ inches long.
 - v. Draw a line x^2+y inches long.
 - vi. Draw a line $7x-y$ inches long.
9. If $x=2\text{cm. } 3\text{mm.}$, draw a line whose length is $5x$.

10. To *bisect* anything is to divide it into two equal parts.

To *bisect* a straight line is to find the point which divides it into *two equal parts*, that is, to find its middle point.

One method of finding the middle point of a straight line is as follows :

Measure the given line with your scale.

Find half its length by dividing its length by two.

Using the scale, mark a point on the line whose distance from one end of the line is equal to half its length.

This point will be the middle point of the line.

11. EXERCISES.

1. Draw a line $2\frac{3}{8}$ in. long and bisect it.
2. Draw a line $5\text{cm. } 4\text{mm.}$ long and bisect it.
3. If $x=2\text{cm. } 4\text{mm.}$, draw a line whose length is $2x$ and bisect it.
4. If $x=2$ and $y=4$, draw a line whose length is $(x+y)\text{cm.}$ and bisect it.

5. If $x = 25\text{cm}$, 8mm , and $y = 10\text{cm}$, 4mm ., draw a line whose length is $(x - y)$ and bisect it.

12. When two straight lines are drawn from the same point they form an *angle*.

In Fig. 3 the two straight lines AB and AC are drawn from the same point A and form the angle BAC.

Definition:—An **angle** is a figure formed by two straight lines drawn from the same point.

The point A (Fig. 3) is the **vertex** of the angle and the lines AB and AC are the **sides** of the angle.

The size of an angle depends only on the *difference in direction* of its sides.

The lengths of the sides have nothing to do with the size of the angle.

If we should prolong the sides AB and AC of Fig. 3 the angle would remain unchanged.

An angle is usually named by three letters, one at the vertex and one at the extremity of each side, as in Fig. 3.

The letter at the vertex is always written between the other two.

An angle is often named by placing a small *italic* letter between the sides and near the vertex. (See Fig. 14, §56.)

13. In Fig. 4 we have the two straight lines AB and CD intersecting at the point E.

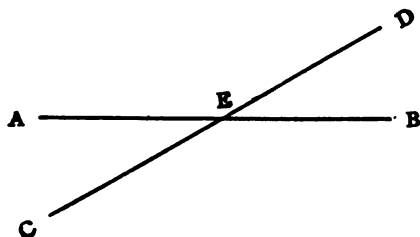


FIG. 4.

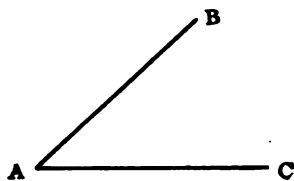


FIG. 3.

How many angles are there in the figure?

Name them.

How many angles are there in Fig. 2?

Name them.

14. Two angles are *equal* when they can be made to *coincide*.

Take your triangles and see whether there are any angles of one equal to any angles of the other.

How many do you find equal?

We shall take up angles more thoroughly further on in the book.

15. Take one of your triangles and see whether any one of its angles will coincide with all four angles of Fig 5.

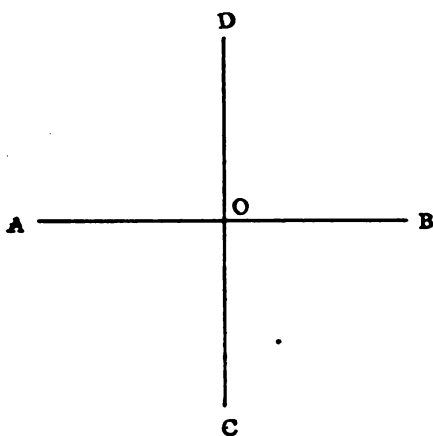


FIG. 5.

Are all four angles equal to the same angle of your triangle?

Are these four angles equal to each other?

Give the reason for your answer.

These four angles are *right angles*.

Definition:—When two straight lines intersect so as to form four equal angles, those angles are **right angles**.

Definition:—Two lines which form a right angle are **perpendicular** to each other.

Is there a right angle on each of your triangles?

How many edges on each of your triangles are perpendicular to each other?

16. If we should suspend a weight from the ceiling by a string, the direction of the string would be **vertical**.

The intersections of the walls of a room with each other are vertical lines.

Mention other vertical lines which you see in the school room.

Straight lines which are parallel to the plane of the horizon are called **horizontal** lines.

The direction of a straight stick floating on still water is horizontal.

The intersections of the walls of a room with the floor are horizontal lines.

Mention other horizontal lines which you see in the school room.

Is a flag staff horizontal or vertical?

Is the top of the railing of a bridge horizontal or vertical?

What kind of an angle does a vertical line make with a horizontal line?

Is a horizontal line perpendicular to a vertical line?

Is a vertical line perpendicular to a horizontal line?

How many right angles are there in the upper corner of a room formed by the intersection of two walls and the ceiling?

17. *A method of drawing perpendicular lines is illustrated by Fig 6.*

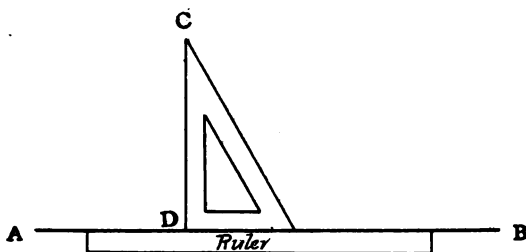


FIG. 6.

The line AB is drawn with a ruler.

Then by placing the triangle against the ruler, as shown in the figure, the line CD is drawn.

CD is perpendicular to AB because the angle of the triangle at D is a right angle.

18.**EXERCISES.**

1. Draw a line from left to right on your paper $2\frac{1}{2}$ in. long. Mark its extremities A and B.

Divide this line AB into parts $\frac{1}{2}$ in. long.

Beginning at the left end, number the points 1, 2, 3, 4, 5, and 6.

At the points numbered draw perpendiculars to the line AB equal, respectively, to the lines given in example 1 of §6.

2. Again draw a line from left to right $2\frac{1}{2}$ in. long and divide it as before.

At the points of division draw perpendiculars equal, respectively, to the lines given in example 2 of §6.

3. Draw a line AB from left to right 6cm. 4mm. long.

At the extremities A and B draw perpendiculars AD and BC each 6cm. 4mm. long.

Connect C and D by a straight line.

How many right angles are there in the figure?

How many pairs of perpendicular lines are there?

19. Draw a line AB 4cm. 9mm. long.

Take any point on the paper above this line and mark it C.

From C draw a perpendicular to AB meeting it at D.

Can you draw another line through C perpendicular to AB?

Draw any other line from C to AB and mark the point where it meets AB with the letter E.

Which line is the shorter, the perpendicular or CE?

Can you draw any other line from C to AB shorter than the perpendicular?

Is the perpendicular the shortest line that can be drawn from an external point to a given straight line?

Statement:—

20. In Fig. 7 we have two straight lines AB and CD.

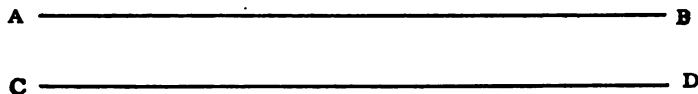


FIG. 7.

Measure the perpendicular distance between these lines at several points.

Is the distance between them *everywhere* the same?

Are these two lines in the same plane?

Do you think that these two lines would meet if you should produce them?

These two lines are called *parallel lines*.

Write a definition of **parallel lines**.

Definition :—

21. A method of drawing parallel lines is illustrated in Fig. 8.

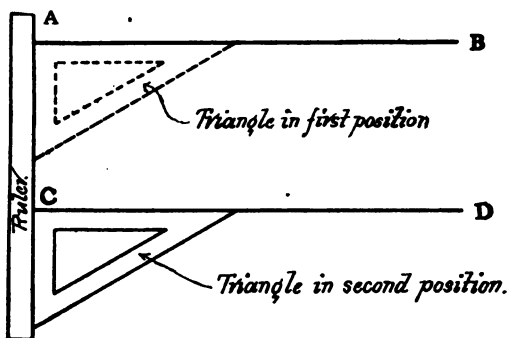


FIG. 8.

To draw a line parallel to the line AB, place a triangle along the line AB as shown in the figure.

Place the ruler against the triangle as shown in the figure and hold it in position firmly.

Now slide the triangle along the ruler until it comes into the desired position and draw the line CD.

Then CD is parallel to AB.

Note:—*This method may be used to determine whether two given straight lines are parallel or not.*

22. Draw a line AB $1\frac{1}{2}$ in. long.

Divide it into two parts $\frac{1}{2}$ in. long.

At the two points of division draw lines perpendicular to AB 5 in. long.

Are these two lines parallel?

Give the reason for your answer.

What general principle may be inferred from this in regard to two straight lines in the same plane perpendicular to the same straight line?

Statement:—

23. Draw two lines of any length parallel to each other.

Mark them AB and CD.

Draw any line perpendicular to CD.

What is the relation of this line to AB?

What general principle may be inferred from this in regard to a straight line perpendicular to one of two parallel lines?

Statement:—

24. Draw a line AB $4\frac{1}{8}$ in. long.

Take any external point and mark it C .

Draw a line through C parallel to AB .

Can you draw another line through C parallel to AB ?

How many lines can be drawn *through an external point parallel to a given straight line*?

Statement:—

25. Problem:—*To divide a line into any number of equal parts.*

Let AB be the given line.

Suppose that we wish to divide the line AB into four equal parts.

Draw any other line from A , such as AC , making an angle with AB .

With your compasses mark off on AC any convenient length four times.

Let these four lengths be AM , MN , NO , and OP .

Draw the line PB .

Through O , N , and M draw lines parallel to PB meeting AB in three points.

These three points will divide AB into four equal parts.

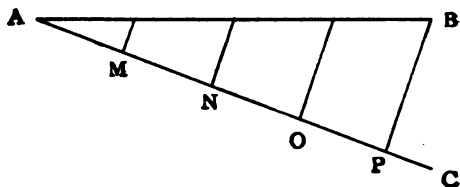


FIG. 9.

26.

EXERCISES.

1. Draw a line $2\frac{7}{8}$ in. long and divide it into five equal parts.

Test the accuracy of the division with your compasses.

2. Divide a line 7cm. 2mm. long into six equal parts.

3. Divide a line 6cm. long into seven equal parts.

4. Divide a line $3\frac{1}{8}$ in. long into four equal parts.

5. Draw a line 11cm. 4mm. long and divide it into five equal parts.

27. In Fig. 10 the line ABCD is a *circumference* of which O is the *centre*.

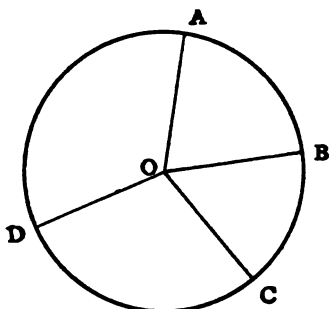


FIG. 10.

Is a circumference a straight line, a broken line, or a curved line?

Measure the distances from the point O to the points A, B, C, and D on the circumference.

Are they all equal?

From the answers to the two preceding questions we can write the following definition of a circumference.

Definition:—A **circumference** is a curved line all points of which are equally distant from a point within called the centre.

28. A circumference incloses or bounds a portion of a plane. This portion of a plane is called a *circle*.

Definition:—A **circle** is a portion of a plane bounded by a circumference.

The point O in Fig. 10 is the centre of the circle as well as of the circumference.

29. The straight lines OA, OB, OC, etc., are *radii* of the circle.

Definition:—A **radius** of a circle is a straight line drawn from the centre to any point on the circumference.

30. The portions of the circumference between A and B, B and C, etc., are called *arcs*.

Definition:—An **arc** is a portion of a circumference. ✓

Name the arcs in Fig. 10.

31. A circumference is drawn with the aid of compasses.

Before drawing a particular circumference, we must know the position of its centre and the length of its radius.

Problem:—*To draw a circumference with a given centre and a given radius.*

Open the compasses so that the distance between its points is equal to the given radius.

Place the needle point on the given centre and draw the circumference with the pencil point.

An arc with given centre and radius is similarly drawn.

32.

EXERCISES.

1. Draw a line AB 1 in. long.

With A as a centre and radius equal to AB describe a circumference.

2. Draw a line AB 5cm. long.

Bisect it and mark its middle point C.

With C as a centre and a radius equal to CB draw a circumference.

With A as a centre and the same radius draw a second circumference.

With B as a centre and the same radius draw a third circumference.

33. Definition:—A straight line which bisects a given straight line and is perpendicular to it is called the **perpendicular bisector** of the given line.

Problem:—*To draw the perpendicular bisector of a given line.*

Let AB (Fig. 11) be the given straight line.

With A as a centre and a radius greater than half of AB describe an arc.

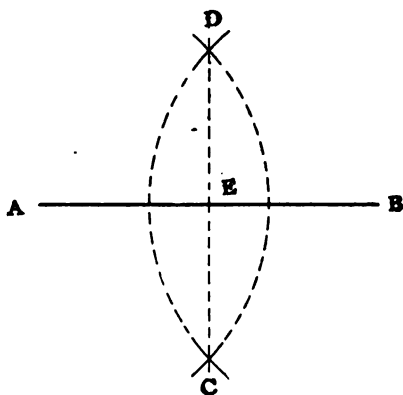


FIG. 11.

Then with B as a centre and the same radius describe a second arc intersecting the first at C and D.

Through C and D draw a straight line intersecting AB at E.

Measure AE and EB.

Does CD bisect AB at E?

Is CD perpendicular to AB? (Test the angles with your triangle.)

Is CD the perpendicular bisector of AB?

In this construction why is it necessary to take a radius greater than half of the given line? (You can answer this by trying to bisect the line by using a radius less than half the given line.)

34.

EXERCISES.

1. Draw a line $3\frac{1}{8}$ in. long.

Draw the perpendicular bisector of this line.

How much should half of this line measure?

See whether you have bisected the line accurately by measuring the two parts of the line.

2. Divide a line 6cm. 3mm. long into four equal parts by two different methods.

3. Take any three points on your paper not in the same straight line and mark them A, B, and C.

Draw the lines AB and BC.

Draw the perpendicular bisectors of AB and BC, and mark the point where they meet D.

With D as a centre and a radius equal to DA draw a circumference.

Does this circumference pass through B and C?

4. See whether you can pass a circumference through three points on the same straight line by the method of the preceding exercise.

Can you do it or not?

Give the reason for your answer.

35. Draw a line AB $2\frac{3}{8}$ in. long.

Draw the perpendicular bisector of AB and mark it CD.

Take any point on CD and see whether it is equidistant from A and B.

(Determine this with your compasses by placing the needle point on the point taken on CD and see whether you can pass an arc through both A and B.)

Do you find it equidistant from A and B?

What general principle may be inferred from this in regard to the position of *every point on the perpendicular bisector of a straight line*.

Statement:—

36. In Fig. 12 we have a circle with the three lines AB, CD, and EF drawn through the centre O. Each of these lines is a *diameter* of the circle.

*Definition:—*A **diameter** of a circle is a straight line passing through the centre and having its extremities in the circumference.

How many diameters is it possible to draw in a circle?

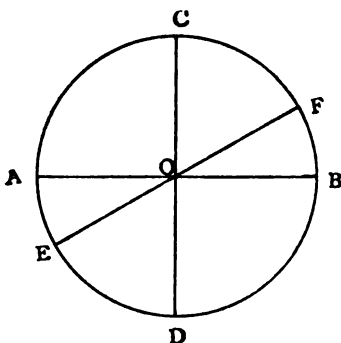


FIG. 12.

37. The diameters in Fig. 12 divide the circle into six parts.

Each of these parts is a *sector*.

Definition:—A **sector** is a portion of a circle bounded by two radii and their intercepted arc.

Name the sectors in Fig. 12.

In Fig. 12 the two diameters AB and CD are drawn perpendicular to each other.

Trace the sector COB on thin

paper.

See whether the tracing will coincide with each of the three sectors AOC, AOD, and BOD.

Are the four sectors AOC, AOD, BOC, and BOD equal to each other?

What part of the circle is each of these sectors?

What part of the circumference is the arc CFB?

What kind of an angle is COB?

A right angle with its vertex at the centre intercepts how much of the circumference?

38. If the circumference was divided into 360 equal parts, how many of these parts would there be in the arc CFB?

If we should draw radii to these points of division on the arc CFB, the angle COB would be divided into how many small angles?

One of these small angles is an *angle of one degree*.

Definition:—An **angle of one degree** is an angle which is $\frac{1}{90}$ of a right angle.

39. *Definition:*—An arc which is $\frac{1}{360}$ of a circumference is an **arc of one degree**.

Remark:—Since the number of degrees in an angle with its

vertex at the centre of a circle and the number of degrees in the intercepted arc are the same, *the angle at the centre is said to be measured by its intercepted arc.*

A degree is subdivided into **minutes** and **seconds**.

The *degree*, *minute*, and *second* are the *units* of angular measure.

The following table gives the relations between these units :

60 seconds (")	equal 1 minute. (')
60 minutes	" 1 degree. (°)
90 degrees	" 1 right angle.

Is the length of an arc of one degree the same for all circles?

To answer this, draw two circles with the same centre but unequal radii and draw an angle at the centre. Are the arcs intercepted on the circumferences equal?

40.

EXERCISES.

1. How many degrees are there in the angle COB of Fig. 12? In the angle AOC? In the angle AOD? In the angle DOB?

What is the sum of the degrees in the angles COB and COA?

What is the sum of the degrees in the angles COB, COA, and AOD?

What is the sum of the degrees in all six angles which have their vertices at the point O?

2. How many degrees are there in 1 right angle? In 2 right angles? In 3 right angles? In 4 right angles? In $\frac{1}{2}$ of a right angle? In $\frac{1}{3}$ of a right angle? In $\frac{2}{3}$ of a right angle?

3. What part of a right angle is an angle of 30° ? Of 60° ? Of 45° ? Of $22^\circ 30'$?

4. How many angles of 90° can be drawn with their vertices at the same point and so that no two angles overlap?

How many of 45° ? Of 30° ? Of 60° ? Of 15° ?

5. How many degrees are there in the angle made by the hands of a clock at 3 o'clock? At 4 o'clock? At 6 o'clock? At 9 o'clock? At 12 o'clock?

6. Through how many degrees does the minute hand of a clock travel between 12 o'clock and quarter past?

7. Through how many degrees does the hour hand of a clock travel between 1 and 5 o'clock?

8. Through how many degrees does the minute hand of a watch travel in a minute of time? In five minutes of time?

9. Draw any angle on a piece of tracing paper and place it over the face of your watch so that the vertex of the angle is at the centre of the face.

How many minute divisions are included between the sides of the angle?

Then how many degrees are there in the angle?

10. Through how many degrees does the hour hand of your watch travel between 1 o'clock and quarter past? Between 2 o'clock and half past? Between 2 o'clock and 5 o'clock?

11. Draw an outline of each of your triangles on tracing paper and determine the number of degrees in each angle as near as you can with your watch.

12. Mention the exact hours when the hands of a clock form a right angle.

13. How many degrees does the hour hand describe in 1 hour? In 2 hours? In 5 hours?

41. Take any point on your paper and mark it O.

From O draw any three straight lines OA, OB, and OC.

What is the sum of the degrees in the three angles AOB, BOC, and COA?

Draw a line OD between OA and OB, a line OE between OB and OC, and a line OF between OC and OA.

How many angles are there now about the point O?

Is the sum of the degrees in all these angles the same as the sum of the degrees in the three angles AOB, BOC, and COA?

To how many right angles is this sum equivalent?

To what is the *sum of all the angles about a point in a plane* equal?

Statement :—

42. Just as we determine the length of a straight line by measuring it, so we determine the size of an angle by measuring it.

To *measure an angle* is to determine the *number of degrees* which it contains.

We measure a straight line with a scale, but an angle is measured by means of a **protractor**.

The accompanying figure shows one form of a protractor. The outer edge ABD is a semi-circumference (half circumference) of which C is the centre and the straight line AD the diameter.

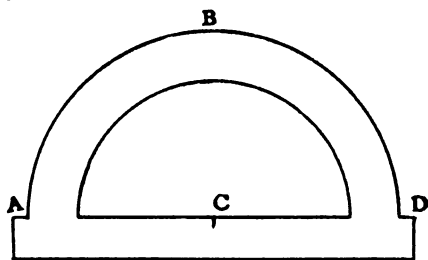


FIG. 13.

The semi-circumference is divided into 180 equal parts.

Each part, therefore, is $\frac{1}{180}$ of the whole circumference or one degree of arc.

To measure an angle, place the protractor on the angle so that the centre C of the protractor is at the vertex of the angle and the edge CD will lie along one side of the angle.

The number of divisions on the arc of the protractor, from

the point D to the point where the other side of the angle crosses the arc, is the number of degrees in the angle.

43.**EXERCISES.**

1. Draw an outline of each of your triangles on paper.

Measure each of the angles with your protractor and mark each with the number of degrees which it contains.

Do you see why one of your triangles is called a $30^\circ \times 60^\circ$ triangle and the other a 45° triangle?

2. From the preceding exercise it is seen that the triangles can be used to draw lines making an angle of 30° , 45° , 60° , or 90° with each other.

Draw a line AB 3 in. long.

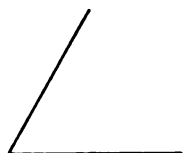
Place the edge of the ruler along this line and placing the triangle against the ruler, draw a line AC making an angle of 60° with AB.

3. Draw a line AB $2\frac{1}{8}$ in. long.

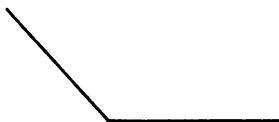
Draw a line AC making an angle of 30° with AB.

4. Draw a line AB $2\frac{1}{2}$ in. long.

Draw a line AC making an angle of 45° with AB.



ACUTE ANGLE.



OBTUSE ANGLE.

44. Definition:—An *acute* (sharp) angle is an angle which is less than a right angle.

Definition:—An *obtuse* (dull) angle is an angle which is greater than a right angle.

Both *acute* and *obtuse* angles are called **oblique angles** as their sides are *oblique* to each other.

45. EXERCISES.

1. Are there more or less than 90° in every acute angle?

Are there more or less than 90° in every obtuse angle?

2. Tell whether the angles containing the following number of degrees are acute, right, or obtuse: 10° , 91° , 125° , 89° , 179° , 45° , 30° , 60° .

3. What kind of an angle is the sum of the two angles containing $39^\circ 20' 30''$ and $49^\circ 39' 30''$?

4. What kind of an angle do the hands of a clock form between 12 o'clock and quarter past?

5. The number of degrees in a certain angle is equal to the difference between $175^\circ 15' 36''$ and $115^\circ 5' 26''$. What kind of an angle is it?

6. Mention the exact hours when the hands of a clock form an acute angle.

7. Mention the exact hours when the hands of a clock form an obtuse angle.

46. Draw any angle and mark it ABC.

From B draw a line BD between the sides BA and BC.

The two angles ABD and DBC are called *adjacent angles*.

Have these two angles the same vertex?

Have they a common side?

From the answers to these two questions write a definition of **adjacent angles**.

Definition :—

The angle ABC is called the *sum* of the angles ABD and DBC.

47. Draw a right angle ABC.

From B draw a line BD dividing the angle ABC into the two angles ABD and DBC.

Definition :—Two angles are called **complementary angles** when their sum is equal to one right angle.

Each angle is called the *complement* of the other.

Are the angles ABD and DBC complementary?

Since the angles ABD and DBC are both complementary and adjacent they are called *complementary adjacent angles*.

Definition :—**Complementary adjacent angles** are two adjacent angles whose exterior (outside) sides are *perpendicular* to each other.

48. Draw a line 4 in. long and mark it AB.

Mark its middle point C.

From C draw any line CD 2 in. long.

Are the angles ACD and DCB adjacent?

Measure each of these angles with the protractor and mark each with the number of degrees which it contains.

What is the sum of the degrees in the two angles?

Their sum is equal to how many right angles?

Definition :—Two angles are called **supplementary angles** when their sum is equal to two right angles.

Each angle is the *supplement* of the other.

Are the angles ACD and DCB supplementary?

Since they are supplementary and adjacent what compound name can you give to them?

Definition :—**Supplementary adjacent angles** are two adjacent angles whose exterior sides form one straight line.

49.

EXERCISES.

1. One of two adjacent angles contains 25° and the other 35° .

Are they complementary adjacent?

What kind of an angle is their sum?

2. One of the two adjacent angles is 30° and the other is 60° .

Are they complementary?

3. How many degrees are there in the complement of each of the following angles?

15° , 30° , 45° , 60° , $54^\circ 43'$, $37^\circ 27' 20''$.

4. How many degrees are there in the supplement of each of the following angles?

135° , 120° , 140° , 150° , 90° , $130^\circ 25' 45''$, $65^\circ 19' 21''$.

5. If $x = 50^\circ$ and $x + y = 90^\circ$, what is the value of y ?

6. If $x = 80^\circ 20'$ what must be the value of y in order that x and y may be complementary? In order that they may be supplementary?

7. One of two complementary angles is twice the other; what is the number of degrees in each?

Solution :—

Let x = the number of degrees in one,

then $2x$ = “ “ “ the other.

Therefore $3x = 90^\circ$

$\therefore x = 30^\circ$ and $2x = 60^\circ$. 30° and 60° . Ans.

8. One of two complementary angles is four times the other; what is the number of degrees in each?

9. One of two complementary angles is five times the other; what is the number of degrees in each?

10. One of two complementary angles is eight times the other; what is the number of degrees in each?

11. One of two supplementary angles is twice the other; what is the number of degrees in each?

12. One of two supplementary angles is five times the other; what is the number of degrees in each?

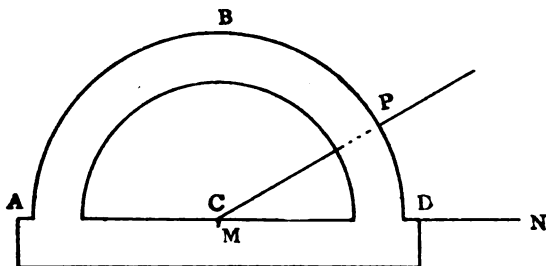
13. One of two supplementary angles is nine times the other; what is the number of degrees in each?

14. One of two supplementary angles is eight times the other; what is the number of degrees in each?

50. Problem:—*To construct an angle of a given number of degrees.*

The method may be best explained by an exercise.

Suppose that we wish to construct an angle of 30° .



Draw any line whatever and mark it MN.

Place the protractor upon MN so that the centre C of the protractor is at M and the radius CD lies along MN.

Read along the arc of the protractor from D to the division marked 30.

Mark a point on the paper opposite this division and letter it P.

Draw a straight line MP.

The angle PMN is an angle of 30°

51.

EXERCISES.

1. With your protractor construct an angle of 75° . Of 120° . Of 45° . Of 150° .

2. Construct a right angle with the protractor.

3. Construct an angle equal to $\frac{1}{3}$ of a right angle. To $\frac{2}{3}$ of right angle. To $\frac{1}{2}$ of a right angle.

4. Construct two complementary adjacent angles one of which shall contain 20° .

5. Construct two complementary adjacent angles one of which shall contain 35° .

6. Construct two supplementary adjacent angles one of which shall contain 85° .

7. Construct two supplementary adjacent angles one of which shall contain 135° .

52. Construct an angle of 35° and mark it ABC.

Below the angle draw a line MN parallel to BA, 3 in. long.

Mark the middle point of MN with the letter O.

Through O draw a line PR parallel to BC, making OP and OR each $1\frac{1}{2}$ in. long.

Measure the four angles formed by the lines MN and PR and mark each angle with the number of degrees which it contains.

Are any of these four angles equal to the angle ABC?

If so, name them.

Are any of these four angles supplements of the angle ABC?

If so, name them.

Are the sides of each of the four angles parallel to the sides of ABC?

What general principle may be inferred in regard to *two angles whose sides are parallel each to each*?

Statement:—

53. Construct an angle ABC of 65° .

Draw a line MN perpendicular to BA and $2\frac{1}{2}$ in. long.

Through O, the middle point of MN, draw a line PR perpendicular to BC, making OP and OR each $1\frac{3}{4}$ in. long.

Measure the four angles formed by the lines MN and PR and mark each with the number of degrees which it contains.

Are any of these four angles equal to ABC?

If so, name them.

Are any of these four angles supplementary to $\angle ABC$?

If so, name them.

Are the sides of each of these angles perpendicular to the sides of $\triangle ABC$?

What general principle may be inferred in regard to *two angles whose sides are perpendicular each to each*?

Statement :—

54. Construct an angle $\angle AOC$ of 55° , making the sides OA and OC 5cm. 5mm. long.

Produce AO to B making OB equal to OA .

“ CO “ D “ OD “ OC .

The angles $\angle AOC$ and $\angle BOD$ are *vertical angles*.

Have these two angles the *same vertex*?

Are the sides of one the *prolongations* of the sides of the other?

From the answers to these two questions write a definition of **vertical angles**.

Definition :—

55. Are the vertical angles $\angle AOC$ and $\angle BOD$ equal?

Are the angles $\angle AOD$ and $\angle BOC$ vertical angles?

Are they equal?

What do you infer in regard to vertical angles in general?

Statement:—

56. Definition:—A straight line which intersects two or more straight lines is called a **transversal**.

In Fig. 14 we have two straight lines intersected by a third straight line.

Which of these lines is the transversal?

How many angles are there in the figure?

The angles between the lines AB and CD are called **interior angles**.

How many interior angles are there? Name them.

The angles outside the lines AB and CD are called **exterior angles**.

How many exterior angles are there? Name them.

The pairs of angles c and e , and d and f are called **alternate-interior angles**.

The pairs of angles a and g , and b and h are called **alternate-exterior angles**.

The pairs of angles a and e , b and f , d and h , and c and g are called **corresponding angles**.

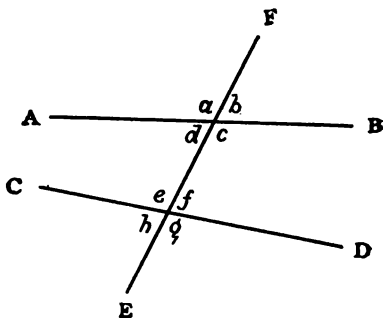


FIG. 14.

57. EXERCISES.

1. If two angles have their sides parallel each to each and

one of them is an angle of 55° , how many degrees may there be in the other?

2. Two angles have their sides parallel each to each and one is acute while the other is obtuse. If there are 96° in one, how many degrees are there in the other?

3. If two angles have their sides perpendicular each to each and one is twice the other, how many degrees are there in each?

4. Draw a figure on your paper similar to Fig 14, making the angle b equal to 30° and the angle f equal to 45° .

Mark the angles a, b, c , etc., as in Fig. 14.

Also mark in each angle the number of degrees which it contains.

How many different sized angles are there in the figure?

How many pairs of *supplementary adjacent* angles are there? Name them.

How many pairs of *vertical* angles are there? Name them.

5. Draw two lines AB and CD extending up and down on your paper and intersect them by a transversal EF.

Mark the angles in the figure a, b, c , etc.

Name the pairs of *alternate-interior* angles.

Name the pairs of *corresponding* angles.

Name the pairs of *alternate-exterior* angles.

58. Draw two parallel lines AB and CD, making them $\frac{3}{4}$ in. apart.

Draw a transversal EF, making an angle of 60° with AB.

Measure all the angles in the figure and mark each with the number of degrees which it contains.

What may be inferred from the figure in regard to the *alternate-interior angles of parallel lines cut by a transversal*?

Statement :—

59. What may be inferred from the figure in regard to the corresponding angles of parallel lines cut by a transversal?

Statement:—

60. What may be inferred from the figure in regard to the two interior angles of parallel lines on the same side of the transversal?

Statement:—

61. Problem:—*To bisect a given angle.*

Let ABC (Fig. 15) be the given angle.

With B as a centre and any convenient radius, describe an arc intersecting the sides of the given angle at E and F.

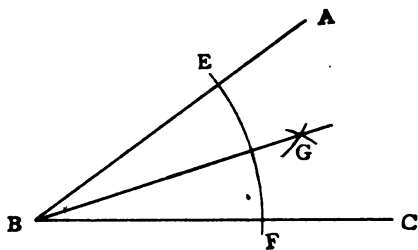


FIG. 15.

Then with E and F as centres and a radius greater than half the distance from E to F, draw two arcs intersecting at G.

Draw a straight line from B to G.

Then the line BG bisects the angle ABC.

62.**EXERCISES.**

1. Draw an outline of your triangles on paper and bisect each of the angles by the method of §61.

Measure the two parts of each angle with the protractor and see whether you have bisected them accurately.

2. Draw an angle of 88° and divide it into four equal parts by bisecting it and then bisecting each half.

3. Suggest a method of dividing an angle into eight equal parts.

4. Into what series of parts can you divide an angle by repeated bisection?

63. Draw an angle of 76° and mark it ABC.

Bisect it by the method of §61.

Mark the extremity of the bisector D.

Measure the angle DBC with the protractor and see whether you have bisected the angle ABC accurately.

Take any point on the bisector BD and mark it P.

Draw perpendiculars from P to BA and BC.

Determine whether these perpendiculars are equal by using your dividers.

Then is P equidistant from BA and BC?

What general principal may be inferred from this in regard to the position of *every point on the bisector of an angle*?

Statement :—

64. Problem :—*To construct an angle equal to a given angle.*

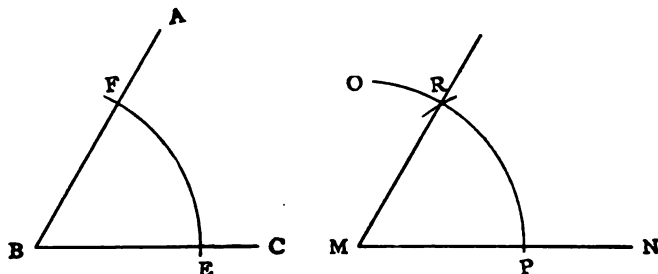


FIG. 16.

Let the angle ABC (Fig. 16) be the given angle.

Draw the line MN of any convenient length.

With B as a centre and any convenient radius draw an arc intersecting the sides of the angle at E and F.

With M as a centre and the same radius draw an arc OP.

Now with P as a centre and a radius equal to EF draw an arc intersecting PO at R.

Draw the straight line MR.

The angle RMN is equal to the angle ABC.

65.

EXERCISES.

1. Using your $30^\circ \times 60^\circ$ triangle draw an angle of 30° .

Now construct an angle equal to this angle by the method of §64.

2. Again using your triangle draw an angle of 60° .

Construct an angle equal to this angle by the method of §64.

3. Using your 45° triangle draw an angle of 45° .

Construct an angle equal to this angle by the method of §64.

4. Construct an angle of 120° with the protractor and then construct a second angle equal to it by the method of §64.

66. Problem:—*To construct an angle equal to the sum of two or more given angles.*

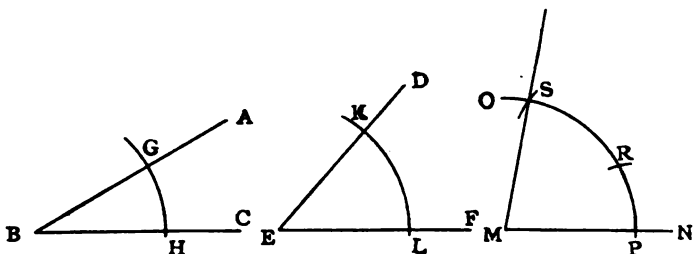


FIG. 17.

Let the angles ABC and DEF be the given angles.

Draw any line such as MN .

With any convenient radius and using B , E , and M in succession as centres, draw the arcs HG , LK , and PO .

Now with P as a centre and a radius equal to GH , draw an arc intersecting PO at R .

With R as a centre and a radius equal to KL , draw an arc intersecting RO at S .

Draw the line MS .

The angle SMN is equal to the sum of the angles ABC and DEF .

Similarly we can construct an angle equal to the sum of three or more given angles.

67.

EXERCISES.

1. Using your triangles, draw three angles of 30° , 45° , and 60° , respectively.

Mark them a , b , and c .

By the method of §66 construct three angles, one equal to the sum of a and b , the second equal to the sum of b and c , and the third equal to the sum of c and a .

2. With the aid of your protractor construct three angles of 12° , 28° , and 50° , respectively.

Mark them a , b , and c .

By the method of §66 construct three angles, one equal to the sum of a and b , the second equal to the sum of b and c , and the third equal to the sum of c and a .

Test the accuracy of your construction with the protractor.

CHAPTER II.

TRIANGLES.

68. In §28 we learned the definition of a circle as *a portion of a plane bounded by a circumference.*

Fig. 18 represents a *triangle*.

Is a triangle a *portion of a plane*?

By what is it *bounded*?

Write a definition of a **triangle** similar to the definition of a circle.

Definition :—

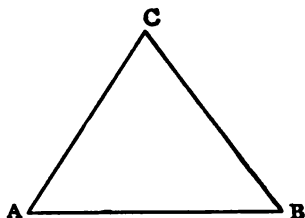


FIG. 18.

The three straight lines AB, BC, and CA are the **sides** of the triangle.

The angles formed by the sides are the **angles of the triangle**.

The vertices of the angles are the **vertices of the triangle**.

The angle C is said to be *opposite* the side AB and the side AB is said to be *opposite* the angle C.

What side is opposite the angle A? the angle B?

What angle is opposite the side BC? the side CA?

The side on which the triangle is supposed to stand is called the **base** of the triangle.

In Fig. 18 the side AB is the *base* of the triangle ABC.

The vertex opposite the base is called the **vertex** of the triangle.

The sides and angles of a triangle are called its **parts**.

How many *parts* has a triangle?

Name the parts of the triangle ABC in order, going around the triangle clock-wise.

The sum of the sides of a triangle is called its **perimeter**.

69. Problem:—*To construct a triangle when the lengths of its three sides are given.*

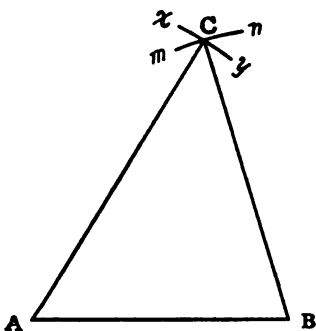


FIG. 19.

Let the lengths of the three sides AB, BC, and CA be 3cm. 5mm., 4cm., and 4cm. 5mm., respectively.

Draw the line AB (Fig. 19) 3cm. 5mm. long.

With B as a centre and a radius of 4cm. describe the arc *mn*.

With A as a centre and a radius of 4cm. 5mm., describe the arc *xy* intersecting *mn* at the point C.

Draw AC and BC.

ABC is the required triangle.

70. Draw a triangle ABC with the lengths of its sides AB, BC, and CA equal, respectively, to 3 in., 2 in., and $1\frac{1}{2}$ in.

Are any two sides of this triangle equal to each other?

A triangle with this relation between its sides is called a *scalene triangle*.

Write a definition of a **scalene triangle**.

Definition:—

71. Draw a triangle ABC with its sides AB, BC, and CA equal, respectively, to $1\frac{1}{2}$ in., $1\frac{5}{8}$ in., and $1\frac{5}{8}$ in.

How many sides of this triangle are equal to each other?

A triangle with this relation between its sides is called an *isosceles triangle*.

Write a definition of an *isosceles triangle*.

Definition :—

The *equal sides* of an isosceles triangle are called its **legs**, the third side is its **base**.

72. Draw a triangle ABC with each of its sides AB, BC, and CA equal to $1\frac{7}{8}$ in.

This is an *equilateral triangle*.

Write a definition of an *equilateral triangle*.

Definition :—

73. Draw a triangle ABC with its sides AB, BC, and CA equal, respectively, to 8cm. 2mm., 7cm. 4mm., and 6cm.

What kind of a triangle is this?

Measure its angles and mark each with the number of degrees which it contains.

Are any two of its angles equal?

Is the largest angle opposite the longest side?

What may be inferred from this in regard to the position of *the largest angle of a scalene triangle* with reference to the sides?

Statement:—

74. Draw a triangle ABC with the side AB equal to 6cm. 6mm., and the sides BC and CA each equal to 5cm. 5mm.

What kind of a triangle is this?

Measure the angles of this triangle and mark each with the number of degrees which it contains.

What general principle may be inferred from the figure in regard to the relation of *the angles opposite the equal sides of an isosceles triangle*?

Statement:—

75. Draw a triangle ABC with its three sides AB, BC, and CA each equal to 4cm. 9mm.

What kind of a triangle is this?

Measure each of its angles and mark each with the number of degrees which it contains.

Is this triangle *equiangular*?

What may be inferred from this in regard to an *equilateral triangle*?

Statement :—

76. Draw a triangle ABC with its sides AB, BC, and CA equal, respectively, to $4\frac{1}{2}$ in., $3\frac{1}{2}$ in., and 3 in.

Produce AB to D making BD 2 in. long.

The angle CBD is called an *exterior* angle of the triangle.

Write a definition of an **exterior angle of a triangle**.

Definition :—

The angles A and C of the triangle are said to be *opposite* the exterior angle CBD.

77. Using the figure you have just drawn, draw a line BE from B parallel to AC.

Measure all the angles in the figure and mark each with the number of degrees which it contains.

Is the angle C equal to the angle CBE?

Is the angle A equal to the angle EBD?

Is the exterior angle CBD equal to the sum of the angles A and C?

What general principle may be inferred from this in regard to the relation between the *exterior angle* of a triangle and the *two opposite interior angles*?

Statement :—

78. Draw a triangle ABC with its sides AB, BC, and CA equal, respectively, to 9cm. 9mm., 5cm., and 8cm.

Produce the side AB to D so as to form an exterior angle CBD.

What name is given to two such angles as ABC and CBD?
(See last definition §48.)

What is the sum of these two angles?

The angle CBD is equal to the sum of what two angles?

What is the sum of the three angles A, C, and ABC?

What general principle may be inferred from this in regard to the *sum of the three angles of any triangle*?

Statement :—

79.

EXERCISES.

1. The three sides of a triangle ABC are 3, 4, and 6 yards in length, respectively.

What kind of a triangle is it?

What is the length of the side which has the largest angle opposite it?

What is the length of the perimeter of the triangle?

2. Draw a triangle ABC with its sides AB, BC, and CA equal, respectively, to 6cm., 8cm., and 10cm.

What kind of a triangle is this?

What is the length of its perimeter?

Measure the angles and mark each with the number of degrees which it contains.

What is the sum of the degrees in the three angles?

To how many right angles is this sum equal?

Does this agree with the statement in §78?

3. Two angles of a triangle contain 35° and 45° , respectively.

How many degrees are there in the third angle?

4. In an isosceles triangle the angle included by the equal sides contains 100° .

How many degrees are there in each of the equal angles?

5. Draw a triangle having all its sides 1 in. long.

Draw all the exterior angles that you can.

How many can you draw?

What name is given to two angles situated as the two exterior angles at the same vertex? (See §54.)

What is the length of the perimeter of this triangle?

6. The perimeter of an equilateral triangle is 3 ft. 9 in. What is the length of each side?

7. The perimeter of an isosceles triangle is 5km. 7hm. and the length of the base is 1km. 3hm.

What is the length of each of the equal sides?

8. The three angles A, B, and C of a triangle are 45° , 70° , and 65° , respectively.

How many degrees are there in the exterior angle at the vertex A? At the vertex B? At the vertex C?

9. The angles A and B of a certain triangle ABC each contain 45° .

How many degrees are there in the third angle C?

10. The angle B of a triangle ABC is a right angle, the other two angles A and C are equal; how many degrees are there in each of them?

11. If one angle of the triangle is obtuse, what kind of angles are the other two?

Give the reason for your answer.

How many obtuse angles can a triangle have?

Give the reason for your answer.

12. How many acute angles may a triangle have?

13. The angle A of a triangle ABC contains x degrees, the angle B contains twice as many degrees as the angle A, and the angle C contains three times as many degrees as the angle A.

How many degrees are there in each of the three angles?

14. The angle B of a triangle ABC contains 30° and the angle C is twice the angle A.

How many degrees are there in each of the angles C and A?

15. The perimeter of a scalene triangle is $4\frac{1}{2}$ in. long, one side is 2 in. long and another side is $1\frac{1}{2}$ in. long.

What is the length of the third side?

16. Each of the equal sides of an isosceles triangle is 4m. 3dm. 5cm. long and the perimeter is 14m. 9dm. 4cm. long.

What is the length of the base?

17. One side of an equilateral triangle is 3m. 6dm. 4cm. 3mm. long.

What is the length of its perimeter?

18. The three angles of a triangle each contain 60° .

What is the length of each side if the length of the perimeter is 1dkm. 9m. 6dm. long?

19. In a triangle ABC the angles A and B are each 50° and the side AB is 4m. long.

If the perimeter is 10m. 2dm. 2cm. 2mm. long; what is the length of the sides BC and CA?

80. In the triangle shown in Fig. 20 the side AB is a side of the angle CAB and also of the angle CBA. These two angles are said to be *adjacent* to the side AB, and the side AB is said to be *included* by the angles A and B.

These three parts are spoken of as *two angles and the included side*.

Similarly for the angles B and C and the side BC, and also for the angles C and A and the side CA.

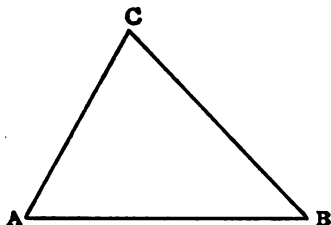


FIG. 20.

81. Problem:—*To construct a triangle when two angles and the included side are given.*

Let the given parts be the side AB, 3 in. long, the angle A equal to 40° and the angle B equal to 30° .

Draw a line AB 3 in. long.

At the point A construct an angle of 40° having AB as one side.

At the point B construct an angle of 30° having BA as one side.

Mark the point where the sides of these two angles intersect C.

Then ABC is the required triangle.

82. Again in the triangle ABC (Fig. 20) the three parts, side AB, angle CAB, and side AC are spoken of as *two sides and the included angle*.

Similarly for the sides BA and BC and the angle ABC, and also for the sides CA and CB and the angle BCA.

83. Problem:—*To construct a triangle when two sides and the included angle are given.*

Let the given parts be AB, 3 in. long; AC, 2 in. long, and the included angle CAB equal to 30° .

Draw a line AB 3 in. long.

At the point A construct an angle CAB equal to 30° , making the side AC 2 in. long.

Connect B and C with a straight line.

ABC is the required triangle.

84.

EXERCISES.

1. Draw a triangle ABC with the side AB $2\frac{1}{4}$ in. long and the angles A and B equal to 35° and 45° , respectively.

How many degrees should there be in the angle C? (See §78.)

Give the reason for your answer.

Measure the angle C and see whether you have drawn the triangle accurately.

2. Draw a triangle ABC with the sides CA and CB each equal to 6cm. and the included angle C equal to 50° .

How many degrees should there be in the sum of the other two angles?

Give the reason for your answer.

Measure these two angles and see whether the figure is correct.

85. Construct a triangle ABC with its sides AB and AC equal, respectively, to 4cm. and 8cm. and the included angle CAB equal to 60° .

Measure the other two angles of this triangle and mark each with the number of degrees which it contains.

How many degrees are there in the angle B?

What kind of an angle is it?

This triangle is a *right triangle*.

Write a definition of a **right triangle**.

Definition :—

The side opposite the right angle of a right triangle is called the **hypotenuse**.

The sides which include the right angle are called the legs of the right triangle.

Which side is the hypotenuse in the triangle you have just drawn?

Name its legs.

86. Draw any right triangle.

Mark the vertex of the right angle A and the other two vertices B and C.

What is the sum of the three angles of this triangle? (See §78.)

Since the angle A is a right angle what is the sum of the angles B and C?

Can either of these angles be a right angle?

Can either of these angles be an obtuse angle?

What kind of an angle must each of them be?

Then how many right angles can a triangle have?

Statement :

87. What is the sum of the two acute angles of a right triangle?

What name is given to two angles which have this sum? (See first definition §47.)

What statement can you make in regard to the two acute angles of a right triangle?

Statement :—

88.**EXERCISES.**

1. One of the acute angles of a right triangle is twice the other; how many degrees are there in each?

2. One of the acute angles of a right triangle is four times the other; how many degrees are there in each?

3. If the angle B of a triangle ABC is a right angle and the angle A is $\frac{1}{4}$ of a right angle, how many degrees are there in the angle C?

4. The two acute angles A and C of a right triangle contain 25° and 65° , respectively; how many degrees are there in the exterior angle at the vertex A? At the vertex B? At the vertex C?

89. Draw a triangle ABC with the angles A and B equal to 54° and 68° , respectively, and the included side AB equal to 7cm. in length.

Draw the bisectors of the angles A and B. (See §61.)

Mark the point where the bisectors intersect O.

Draw a straight line from O to C.

Is OC the bisector of the angle C? (Determine this with the protractor.)

Is O equidistant from AB and AC? (See §63.)

Give the reason for your answer.

Is O equidistant from the sides BA and BC? From the sides CA and CB?

Then is O equidistant from all three sides of the triangle?

What general principle may be inferred from this in regard to the bisectors of the angles of a triangle?

Statement :—

Can the bisectors of the angles of a triangle meet *outside* of the triangle?

Give the reason for your answer.

90. Draw a triangle ABC with the sides AB, BC, and CA equal, respectively, to 7cm. 4mm., 6cm. 3mm., and 5cm. 9mm.

Draw the perpendicular bisectors of the sides AB and BC. (See §33.)

Mark the point where they intersect O.

Draw the perpendicular bisector of CA.

Does the bisector of CA pass through O?

Is O equidistant from the vertices A and B? (See §35.)

Give the reason for your answer.

Is O equidistant from B and C? from C and A?

Then is O equidistant from the three vertices of the triangle?

What general principle may be inferred from this in regard to the *perpendicular bisectors of the sides of a triangle*?

Statement:—

91.

EXERCISES.

1. Draw any *right* triangle ABC.

Draw the perpendicular bisectors of the sides.

Where do the perpendicular bisectors of the sides of a right triangle meet?

2. Draw any *obtuse* angle triangle ABC.

Draw the perpendicular bisectors of the sides.

Where do the perpendicular bisectors of the sides of an obtuse angle triangle meet?

3. By considering the figure drawn for §90 and the two figures of the two preceding exercises, what do you observe in regard to the difference in position of the point of intersection of the perpendicular bisectors in an acute angle triangle, a right triangle, and an obtuse angle triangle?

92. Draw a triangle ABC with its sides AB, BC, and CA equal, respectively, to 3 in., $2\frac{1}{2}$ in., and $2\frac{5}{8}$ in.

Mark the middle point of each of its sides.

Draw a line from each vertex to the middle point of the opposite side.

These three lines are called the *medians* of the triangle.

Write a definition of a **median** of a triangle.

Definition:—

93. What do you observe in regard to the medians of the triangle you have just drawn?

What may be inferred from this in regard to *the medians of a triangle*?

Statement:—

Can the medians of a triangle meet outside the triangle?

Give the reason for your answer.

94. Draw any triangle ABC.

Draw a perpendicular from A to BC, and also from B to CA, and from C to AB.

These three perpendiculars are the *altitudes* of the triangle.

Write a definition of an **altitude** of a triangle.

Definition:—

When we speak of the **altitude** of a triangle we mean the

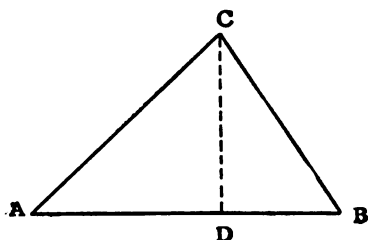


FIG. 21.

altitude which is *perpendicular to the base*. The length of this perpendicular is the height of the triangle.

In Fig. 21 the line CD is the altitude of the triangle.

95. Draw a triangle ABC with the sides AB and AC equal, respectively, to 5cm.

6mm. and 6cm, 5mm. and the included angle A equal to 55° .

Draw the three altitudes.

What do you observe in regard to these three altitudes?

What general principle may be inferred from this in regard to the *three altitudes of a triangle*?

Statement:—

96.**EXERCISES.**

1. Draw any *right* triangle ABC.

Draw the three altitudes of this triangle.

Where do the altitudes of this triangle meet?

2. Draw any *obtuse angle* triangle.

Draw the three altitudes of this triangle.

Where do the three altitudes of this triangle meet?

3. By considering the figure drawn for §95 and the figures of the two preceding exercises, what do you observe in regard to the difference in position of the point of intersection of the altitudes in an *acute angle* triangle, a *right* triangle, and an *obtuse angle* triangle?

97. Draw a triangle ABC with each of the three sides $2\frac{1}{8}$ in. long.

What kind of a triangle is this?

Draw the three altitudes of the triangle.

Are these three lines also the perpendicular bisectors of the sides?

Are they the bisectors of the angles?

Are they the medians of the triangle?

What four sets of lines, may we infer from this, coincide in an equilateral triangle?

Statement :—

98. Draw a triangle ABC with the sides AB, BC, and CA equal, respectively, to 6cm. 4mm., 7cm. 2mm., and 6cm. 8mm.

Bisect the sides with the aid of the scale and mark the middle points M, N, and P, respectively.

Draw the line PN.

What is the direction of PN with reference to AB?

Measure the length of PN.

The length of PN is what part of the length of AB?

What may we infer from this in regard to the relation which *a line joining the middle points of two sides of a triangle* holds to the third side?

Statement :—

Draw the lines MP and MN and see whether this statement is true of them.

99. Draw a triangle ABC with the side AB $2\frac{1}{2}$ in. long and the two adjacent angles, A and B, equal, respectively, to 55° and 75° .

Draw another triangle MNP, on tracing paper, with the side MN $2\frac{1}{2}$ in. long and the two adjacent angles M and N equal, respectively, to 75° and 55° .

See whether the triangle MNP can be made to coincide with the triangle ABC.

Are the two triangles equal?

What general principle may be inferred from this in regard to *two triangles which have one side and the two adjacent angles of one equal, respectively, to one side and the two adjacent angles of the other?*

Statement :—

100. Draw a triangle ABC with the sides AB and AC equal, respectively, to 6cm. 5mm. and 5cm. 8mm. and the included angle A equal to 75° .

Draw another triangle MNP, on tracing paper, with the sides MN and MP equal, respectively, to 5cm. 8mm. and 6cm. 5mm. and the included angle M equal to 75° .

See whether the triangle MNP can be made to coincide with the triangle ABC.

Are the two triangles equal?

What general principle may be inferred from this in regard to *two triangles which have two sides and the included angle of one equal, respectively, to two sides and the included angle of the other?*

Statement:—

101. Draw a triangle ABC with the sides AB, BC, and CA equal, respectively, to 5cm., 6cm., and 7cm.

Draw another triangle MNP, on tracing paper, with the sides MN, NP, and PM equal, respectively, to 7cm., 5cm., and 6cm.

See whether the triangle MNP can be made to coincide with the triangle ABC.

Are the two triangles equal?

What general principle may be inferred from this in regard to *two triangles which have three sides of one equal to three sides of the other?*

Statement:—

102. Draw a triangle ABC with the side AB $1\frac{1}{2}$ in. long, the angle A equal to 50° and the angle B equal to 75° .

Draw another triangle MNP with the side MN equal to $2\frac{1}{2}$ in., the angle M equal to 50° and the angle N equal to 75° .

How many degrees are there in the angle C of the triangle ABC?

How many in the angle P of the triangle MNP?

Have these two triangles three angles of the one equal to three angles of the other?

Are the two triangles equal?

103. From the statements of \$99, \$100, and \$101 we observe that it is necessary to know that two triangles have *how many parts* of one equal to how many parts of the other in order to say they are equal?

From §102 we see that *it is necessary for at least one of these parts to be a side.*

104. The following summary may be found useful in fixing in the mind of the pupil certain facts in regard to triangles.

What is a triangle?

How many sides has a triangle? Ans. **Three.**

How many angles has a triangle? Ans. **Three.**

How many kinds of triangles are there, named with reference to their sides? Ans. **Three.**

What is the **first** kind? Ans. *Scalene*, having **no** sides equal.

What is the **second** kind? Ans. *Isosceles*, having **two** sides equal.

What is the **third** kind? Ans. *Equilateral*, having **three** sides equal.

We must know that two triangles have how many parts of one equal to how many parts of the other in order to say that they are equal? Ans. **Three.**

How many cases of equal triangles have we had? Ans. **Three.**

What are the equal parts in the **first** case? Ans. **One side** and two adjacent angles.

What are the equal parts in the **second** case? Ans. **Two sides** and the included angle.

What are the equal parts in the **third** case? Ans. **Three sides**.

CHAPTER III.

QUADRILATERALS.

105. Fig. 22 represents a *quadrilateral*.

Write a definition of a **quadrilateral** similar to the definition of a triangle, which was written in §68.

Definition :—

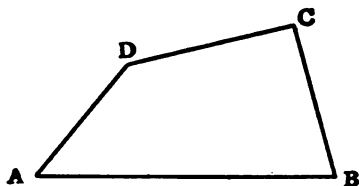


FIG. 22.

The bounding lines are called the **sides** of the quadrilateral.

The angles formed by the sides are the **angles** of the quadrilateral.

The vertices of the angles are the **vertices** of the quadrilateral.

The sum of the sides is the **perimeter** of the quadrilateral.

106. There are three kinds of quadrilaterals named with reference to the number of *parallel sides* which the figure has.

The three kinds are the *trapezium*, the *trapezoid*, and the *parallelogram*.

107. Draw a line AB 7cm. long.

From A draw a line AD 3cm. long, making an angle of 65° with AB.

From B draw a line BC 4cm. long, making an angle of 70° with BA.

Draw a line from D to C.

Is the figure ABCD a quadrilateral?

Are any of its sides parallel?

A quadrilateral of this kind is called a *trapezium*.

Write a definition of a **trapezium**.

Definition :—

108. Draw a line AB $2\frac{3}{4}$ in. long.

From A draw a line AD $1\frac{1}{2}$ in. long, above AB, making an angle of 62° with AB.

From D draw a line parallel to AB.

From B draw a line making an angle of 50° with BA.

Mark the point where these last two lines meet with the letter C.

Is the figure ABCD a quadrilateral?

How many sides of this figure are parallel?

A quadrilateral of this kind is called a *trapezoid*.

Write a definition of a **trapezoid**.

Definition :—

The parallel sides are called the **lower** and **upper bases** of the trapezoid.

The non-parallel sides are called the **legs** of the trapezoid.

Name the bases and legs in the trapezoid you have just drawn.

The **altitude** of a trapezoid is the perpendicular distance between its bases.

109. Draw a line AB $1\frac{7}{8}$ in. long.

From A draw a line AD $\frac{1}{4}$ in. long, above AB, making an angle of 75° with AB.

Draw a line from D parallel to AB.

Draw a line from B parallel to AD.

Mark the point where these two lines meet with the letter C.

Is the figure ABCD a quadrilateral?

Are any of its sides parallel?

If so, which ones?

A *quadrilateral* of this kind is called a *parallelogram*.

Write a definition of a **parallelogram**.

Definition :—

The lines AB and CD are called the **lower** and **upper bases** of the parallelogram.

The **altitude** of a parallelogram is the perpendicular distance between its bases.

110. Parallelograms are divided into two classes, namely, *rhomboids* and *rectangles*, depending upon whether the angles in the figure are *oblique* or *right*.

In the following sections we shall construct a parallelogram of each kind and learn a definition of each.

111. Are the angles in the figure which you drew for §109 right or oblique?

A parallelogram whose angles are of this kind is called a *rhomboid*.

Write a definition of a **rhomboid**.

Definition :—

112. Draw a parallelogram ABCD with the sides AB and AD equal, respectively, to $1\frac{1}{8}$ in. and $1\frac{1}{8}$ in., and the included angle A a right angle.

Are all the angles in this figure right angles?

A parallelogram of this kind is called a *rectangle*.

Write a definition of a **rectangle**.

Definition :—

113. Construct an *equilateral rhomboid* ABCD with its sides 4cm. long.

A rhomboid of this kind is called a *rhombus*.

Write a definition of a **rhombus**.

Definition :—

Remark :—A rhombus is also called a lozenge or diamond shaped figure.

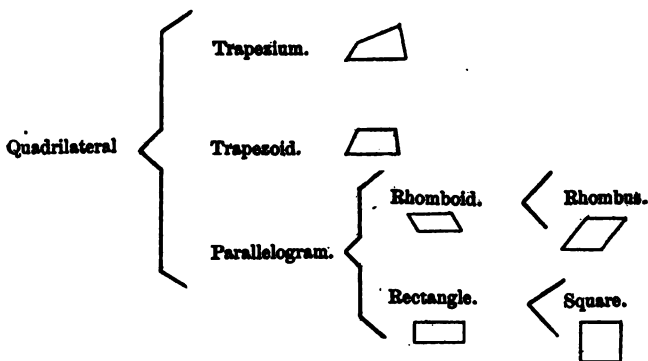
114. Construct an *equilateral rectangle* ABCD with its sides 4cm. long.

A rectangle of this kind is called a *square*.

Write a definition of a **square**.

Definition :—

The following diagram shows the relation between the different kinds of quadrilaterals :



115. Draw any quadrilateral and mark it ABCD.

Draw a straight line from A to C.

What is the sum of the angles of the triangle ABC? Of the triangle DAB? (See §78.)

What is the sum of all the angles in the two triangles?

Is the sum of the angles of the two triangles the same as the sum of the angles of the quadrilateral?

What is the sum of the angles of the quadrilateral?

What may be inferred from this in regard to the sum of the angles of any quadrilateral?

Statement :—

116. Draw a line AB $2\frac{3}{8}$ in. long.

From A draw a line AD $1\frac{1}{4}$ in. long making an angle of 60° with AB.

From D draw a line parallel to AB.

From B draw a line parallel to AD.

Mark the point where these two lines intersect with the letter C.

What is the name of the figure ABCD?

Give the reason for your answer.

Measure the opposite sides AB and DC. Are they equal?

“ “ “ AD “ BC. Are they equal?

What may be inferred from this in regard to *the opposite sides of a parallelogram*?

Statement:—

117. Referring to the figure which you have just drawn, the angles A and C and also the angles B and D are called *opposite angles* of the parallelogram.

Measure all the angles in this parallelogram and mark each with the number of degrees which it contains.

What may be inferred from the figure in regard to *the opposite angles of a parallelogram*?

Statement:—

Two angles of a quadrilateral which have their vertices at the extremities of the same side are called *adjacent angles of the quadrilateral*.

118. Again referring to the figure drawn for §116, what is the relation between the angles A and D? Between the angles D and C?

What may be inferred from the figure in regard to the relation of *two adjacent angles of a parallelogram*?

Statement :—

119. Draw any quadrilateral and mark it ABCD.

Draw a line from the vertex A to the opposite vertex C and also a line from B to D.

These two lines are called the *diagonals* of the quadrilateral.

Write a definition of a **diagonal** of a quadrilateral.

Definition :—

120. Draw any parallelogram ABCD.

Draw its diagonals.

Mark the point where the diagonals intersect O.

Does the point O *bisect* the diagonal AC?

Does it also *bisect* the diagonal BD?

What may be inferred from this in regard to *the diagonals of a parallelogram*?

Statement :—

121. Draw any parallelogram ABCD.

Draw the diagonal AC.

Is AB equal to DC? (See §116.)

Is AD equal to BC?

Is AC a side of both the triangles ABC and ADC?

Are the two triangles equal? (See §101.)

Give the reason for your answer.

Does the diagonal of a parallelogram divide the figure into two equal triangles?

Statement :—

122. Draw a trapezoid ABCD with its lower base AB 7 cm. 5mm. long, its upper base 5cm. 5mm. long, and its altitude 4cm.

Bisect the legs AD and BC, and mark their middle points M and N, respectively.

Draw a line from M to N.

This line MN is called the *median* of the trapezoid.

Write a definition of the **median** of a trapezoid.

Definition :—

123. What is the length of the median in the figure you have just drawn?

What is the sum of the bases?

The length of the median is *what part of the sum of the bases?*

What is the direction of the median with reference to the bases?

What may be inferred in regard to the median of a trapezoid?

Statement:—

124.

EXERCISES.

1. Draw a rhomboid and a rectangle and letter each ABCD, making the sides AB and AD in each $2\frac{1}{2}$ in. and $1\frac{1}{4}$ in. respectively.

Draw the diagonals in each figure.

Are the diagonals in either figure equal?

If so, in which one?

Does the statement of §120 hold good for each of these figures?

2. Draw a rhombus and a square and letter each ABCD, making the sides of each $1\frac{1}{2}$ in. long.

Draw the diagonals in each figure.

Are the diagonals in either figure equal to each other?

If so, in which one?

What is the direction of the diagonals of the square with reference to each other?

Is this also true for the rhombus?

Does the statement of §120 hold good for each of these figures?

3. In what kind of quadrilaterals is it obvious that the sum of its angles is equal to four right angles?

4. If one angle of a parallelogram is equal to 35° , how many degrees are there in each of the other three? (See §117 and §118.)

5. How many obtuse angles may a quadrilateral have?

6. How many right angles may a quadrilateral have?

7. How many acute angles may a quadrilateral have?

8. If three angles of a quadrilateral are $50^\circ 45' 23''$, $60^\circ 27' 32''$ and $45^\circ 15' 17''$, how many degrees, minutes, and seconds are there in the fourth angle?

9. One angle of a quadrilateral contains 75° and the other three are all equal; how many degrees are there in each of the other three angles?

10. Into what kind of triangles does the diagonal of a rectangle divide the figure?

11. The upper base of a trapezoid is 15m. 4dm. long, and the lower base is 20m. 6dm. long; what is the length of the median?

12. One leg of a trapezoid is 3m. 4dm. long, the other leg is half as long again, and the median is 12m. 2dm. 5cm. long; what is the length of the perimeter?

13. The perimeter of a square is 40ft. What is the perimeter of a rectangle formed by placing two of these squares side by side?

14. What is the perimeter of a square formed by placing four squares together, if the perimeter of each square is 8m.?

15. What is the least number of squares that it takes to form a square? What is the next to the least number? The next? etc., etc.

16. The sides AB and AD of a parallelogram ABCD are 11m. 6cm., and 5m. 5mm. in length, respectively, what is the perimeter of the figure?

CHAPTER IV.

CIRCLES.

125. In the following space write the definition of a circle which has been given in §28.

Definition :—

126. In Fig. 23 the line AB is a *chord* of the circle.

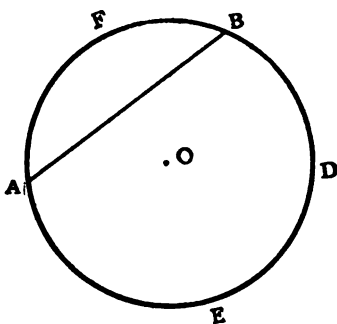


FIG. 23.

What kind of a line is a *chord*?

Its extremities lie in what line?

From the answers to these two questions write a definition of a **chord**.

Definition :—

A chord is said to *subtend* the arc and the arc is *subtended* by the chord. In Fig. 23 the chord AB subtends the arc AFB and the arc AEDB.

127. Draw a circumference with a radius of 1 in. on tracing paper.

Mark the centre O.

Draw a chord passing through O and mark it AB.

What name have we already learned for a chord passing through the centre of a circle? (See §36.)

Fold the figure along the diameter AB and see whether the two parts of the circle on each side of the diameter coincide.

Are the two parts of the circle equal?

Are the two parts of the circumference equal?

What may be inferred from this in regard to the parts into which a *diameter divides the circle and the circumference?*

Statement:—

Remark: Half of a circle is called a **semi-circle** and half of a circumference is called a **semi-circumference**.

128. Draw a circumference with centre O and radius equal to 4cm. 2mm.

Draw a diameter AB.

Draw any other chord AC from the point A.

Which is the longer, AB or AC?

Is it possible to draw any chord longer than the diameter?

Make a statement with regard to the length of a diameter of a circle compared with that of any other chord.

Statement:—

129. Problem :—*To draw a chord of given length in a given circle.*

Draw a circumference with a radius of $1\frac{1}{2}$ in.

Suppose that we wish to draw a chord in this circle $2\frac{1}{2}$ in. long.

Take any point on the circumference and mark it A.

With A as a centre and a radius equal to $2\frac{1}{2}$ in., draw an arc cutting the circumference at a point B.

Draw the chord AB.

Then AB is the required chord.

130. Draw a circumference with centre O and radius equal to 3cm. 5mm.

Draw a chord in this circle 4cm. 4mm. long and mark it AB.

Now draw a diameter perpendicular to the chord. (See §17.)

Mark the point where the diameter intersects the chord C.

Measure CA and CB.

Does the diameter bisect the chord AB?

What may be inferred from this?

Statement:—

131. Draw a circumference with centre O and radius 3cm. 6mm. long.

Draw a chord AB 4cm. 4mm. long.

Draw another chord CD 5cm. 5mm. long.

Draw a perpendicular from the centre O to AB.

Draw a perpendicular from the centre O to CD.

Measure these perpendiculars.

Which chord is at the greater distance from the centre, the longer or the shorter?

What may be inferred from this in regard to the distances of two unequal chords from the centre of a circle?

Statement:—

132. Draw a circumference with centre O and radius 8cm. 7mm. long.

Draw two equal chords AB and CD each 5cm. long.

Draw a perpendicular from O to AB and from O to CD.

Are these perpendiculars equal?

Are the chords equally distant from the centre?

What general principle may be inferred from this?

Statement:—

133.

EXERCISES.

1. The radius of a circle is 7m. 8dm. long; what is the length of the longest chord that can be drawn in this circle?

2. Two chords AB and CD of a certain circle are 9 in. and 15 in. in length, respectively; which chord is at the greater distance from the centre?

3. Draw a circumference with centre O and radius 4cm. 1mm. long.

Mark a point P 2cm. 3mm. from the centre O.

Through the point P draw the longest and the shortest chords possible.

What direction have these chords with reference to each other?

What is the position of the point P on the shortest chord?

134. Draw a circumference with any radius.

From any point B in the circumference, draw two chords BA and BC, forming the angle ABC.

This angle is called an *inscribed angle*.

Write a definition of an **inscribed angle** which describes the *position of the vertex* and the *character of its sides*.

Definition :—

The arc AC which lies between the sides of the angle is said to be **intercepted** by the angle.

135. Draw a circumference with centre O and radius 5cm. long.

From any point B in the circumference, draw a chord BA 9cm. long.

Draw another chord BC from B, making an angle of 40° with BA.

Draw the radii OA and OC.

Measure the angle AOC, and mark it with the number of degrees which it contains.

The angle ABC is what fractional part of the angle AOC?

Do both of these angles intercept the same arc?

Which of these angles is measured by the arc? (See remark, §39.)

By what part of the arc is the inscribed angle ABC measured?

Give the reason for your answer.

What relation between *an inscribed angle* and *its intercepted arc* may be inferred from this?

Statement :—

136. Draw a circumference with any radius.

Draw a diameter AB.

Take any point on either semi-circumference and mark it C.

Draw CA and CB.

Is the angle ACB an inscribed angle? (See §134.)

Name its intercepted arc.

What part of the circumference is its intercepted arc?

How many degrees are there in it?

Then how many degrees are there in the angle ACB? (See §135.)

Give the reason for your answer.

What kind of an angle is it?

What may be inferred from this in regard to *an angle inscribed in a semi-circle*?

Statement :—

137. Draw a circumference with any radius.

Draw two chords AB and CD so as to intersect each other.

Mark their point of intersection O.

What is the relation between the angles AOC and DOB?
(See §54.)

What arcs are intercepted by the angle AOC and its vertical angle DOB?

What arcs are intercepted by the angle AOD and its vertical angle COB?

138. Draw the chord CB in the figure just constructed for §137.

What kind of an angle is AOC with reference to the triangle OCB? (See §76.)

This angle is equal to the sum of what two angles? (See §77.)

By what is the angle OCB *measured*? (See §135.)

By what is the angle OBC *measured*?

Since the angle AOC is equal to the sum of these two angles, by what is it measured?

What may be inferred from this in regard to *the measure of an angle formed by the intersection of two chords*?

Statement:—

By what is the angle AOD measured?

139.

EXERCISES.

1. An inscribed angle intercepts an arc of 84° ; how many degrees are there in the angle?

2. If an inscribed angle contains 35° , how many degrees are there in the intercepted arc?

3. What fractional part of the circumference does an inscribed angle of 15° intercept?

4. If an inscribed angle contains 20° , what fractional part of the circumference is its intercepted arc?

5. If the circumference of a circle is 150 yds., what is the length of an arc intercepted by an inscribed angle of 30° ?

6. An inscribed angle and an angle at the centre both intercept the same arc which is $\frac{1}{10}$ of the circumference; how many degrees are there in each of them?

7. If the circumference of a circle is 100 ft., how many degrees are there in an inscribed angle which intercepts an arc of 10 ft.?

8. How many inscribed angles may be drawn in a circle so as to intercept the same arc?

9. How many degrees are there in an inscribed angle which intercepts $\frac{1}{4}$ of the circumference?

10. One pair of vertical angles formed by two intersecting chords, intercept two arcs which are, respectively, $\frac{1}{3}$ and $\frac{1}{4}$ of the circumference; how many degrees are there in each of these angles?

How many degrees are there in each of the other pair of angles?

11. The sum of the two arcs intercepted by two vertical angles formed by two chords intersecting within the circumference is equal to $\frac{1}{3}$ of the circumference; how many degrees are there in the angle?

12. An angle formed by two chords intersecting within the circumference contains 75° ; what part of the circumference is the sum of the arcs intercepted by the angle and its vertical?

140. Draw a circumference with any radius.

Inscribe any quadrilateral in this circle.

Mark its vertices A, B, C, and D.

What arc is intercepted by the angle A?

What arc is intercepted by the opposite angle C?

How many degrees are there in the sum of these two arcs?

Then how many degrees are there in the sum of the two opposite angles A and C?

What is the relation between these two angles? (See §48.)

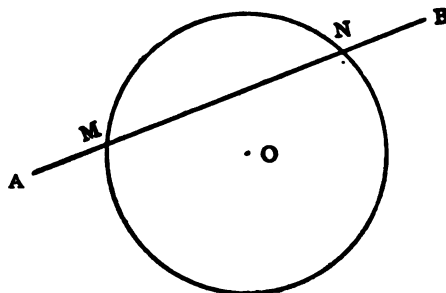
Have the two opposite angles B and D the same relation?

Explain the reason for your answer.

What may be inferred in regard to *the opposite angles of an inscribed quadrilateral*?

Statement :—

141. In Fig. 24 we have a circumference intersected by a straight line AB.



The line AB is a *secant*.

In *how many points* does a secant intersect the circumference?

Write a definition of a secant.

FIG. 24.

Definition :—

What is the difference between a secant and a chord?

Is a secant a part of a chord, or is a chord a part of a secant?

Is there a chord in Fig. 24? If so, name it.

142.**EXERCISES.**

1. Draw a ~~circumference~~ with centre O and radius $1\frac{1}{2}$ in. long.

Draw a secant to this circle so that the chord will be $1\frac{1}{2}$ in. long. (See §129.)

2. Draw a circumference with centre O and radius 3cm. 5mm. long.

Draw a secant 9cm. 6mm. long, and so situated that it is trisected (divided into three equal parts) by the points where it intersects the circumference.

143. Draw a circumference with centre O and radius 3cm. 3mm. long.

Take any point outside the circle, 6cm. 7mm. from the centre, and mark it P.

From P draw a secant through the centre O.

Mark the points where it intersects the circumference A and B, the point A being the nearer to P.

Remark:—The distance from the external point P to the second point of intersection B is called the length of the secant, and the segment of the secant PA is called the external segment of the secant.

What is the length of the secant in the figure just drawn?

What is the length of the external segment?

144. Draw a circumference with any radius and mark its centre O.

Take any external point and mark it P.

From P draw a secant passing through the centre O.

Mark the points of intersection A and B, the point A being the nearer to P.

Draw any other secant from P and mark the points of intersection C and D.

Draw the radius OD with a dotted line.

Which is the greater, the broken line $PO + OD$ or the straight line PD ? (See axiom §2.)

. Give the reason for your answer.

Is the broken line $PO + OD$ equal to the straight line $PO + OB$?

Give the reason for your answer.

. Which is the greater the secant PB or the secant PD ?

Then the longest secant that can be drawn to a circle from an external point passes through what fixed point?

Statement.—

145. In Fig. 25 we have a circumference and the parallel secants AB , CD , EF , etc.

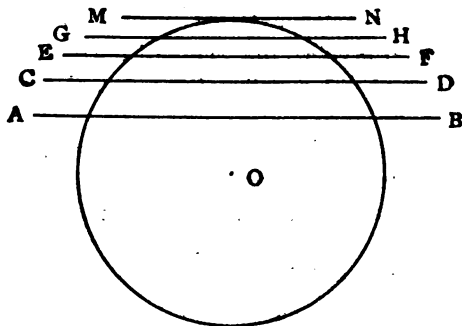


FIG. 25.

Measure the distance in metric units between the points where the secant AB intersects the circumference.

Do the same for each of the other secants.

Beginning with AB and taking the secants in order,

does the distance between the points of intersection increase or decrease as we go from the centre?

Is it possible to draw a line so that the distance between the points where it intersects the circumference is zero?

Is there any such line in the figure?

If so, name it.

Does this line pass through the circumference or simply touch it?

How many points has it in common with the circumference?

Such a line as this is called a *tangent* to the circle.

Write a definition of a **tangent**.

Definition :—

The point where the tangent touches the circumference is called the **point of contact** or **point of tangency**.

146. Draw a circumference with centre O and radius 4cm. 6mm. long.

Take any point A on the circumference and draw the radius OA.

Through the point A draw a line perpendicular to OA.

Mark this line BC.

Is BC a tangent to this circle?

What may be inferred from this in regard to a *straight line perpendicular to a radius at its extremity*?

Statement :—

147. What method for drawing a tangent to a given circle at a given point on the circumference is suggested by the preceding section?

Write out this method below.

Problem:—*To draw a tangent to a given circle at a given point on the circumference.*

148. Problem:—*To draw a tangent to a given circle from a given external point.*

Draw a circumference with centre O and any radius.

Take any external point P.

Draw a straight line from P to O.

Find the middle point of PO and mark it C.

With C as the centre draw a circumference having PO for a diameter.

Mark the points where this circumference intersects the given circumference M and N.

Draw a line from P through M and this will be the required tangent.

This construction is based on §146 as can be seen from the following:

Draw the radius OM.

What kind of an angle is OMP? (See §136.)

Is the line PM tangent to the given circle?

Give the reason for your answer.

Another tangent can be drawn from P through N.

149. Draw a circle with centre O and radius 3cm.

Take an external point P 5cm. from the centre.

Draw a line from P to O.

Draw two tangents to the circle from the point P.

Remark:—The length of a tangent drawn from an external point to a circle is the distance from the external point to the point of contact.

Measure the two tangents you have just drawn.

Measure the angles which they make with the line PO.

What may be inferred from this in regard to *two tangents drawn from an external point to a circle*?

Statement:—

150.

EXERCISES.

1. Draw a circumference with centre O and radius $1\frac{3}{8}$ in. long.

Take any point A on the circumference and draw a tangent at that point $2\frac{1}{4}$ in. long, and so situated that it will be bisected at A.

2. Draw a circumference with centre O and radius 3cm. 1mm. long.

From an external point P, 6cm. 2mm. from the centre, draw two tangents to the circle.

151. Draw a circumference with any radius.

Draw a diameter AB.

At the point A draw a tangent CD.

Draw any chord AE within the angle CAB.

What kind of an angle is CAB?

How many degrees are there in the arc AEB?

Then the angle CAB is measured by what part of the arc AEB?

By what is the angle EAB measured? (See §135.)

Is the angle CAE equal to the angle CAB minus the angle EAB?

Then by what is the angle CAE measured?

What may be inferred from this in regard to the measure of an angle formed by a tangent and a chord?

Statement:—

152. Draw a circumference with any radius.

Take any external point and mark it P.

From P draw any two secants, one of them intersecting the circumference in the points A and B and the other intersecting the circumference in the points C and D.

Draw the chord AD.

Is the angle BAD equal to the sum of the angles P and ADC? (See §77.)

Give the reason for your answer.

Then is the angle P equal to the angle BAD minus the angle ADC?

By what is the angle BAD measured? (See §135.)

By what is the angle ADC measured?

Since P is equal to the difference of these two angles, by what is it measured?

What may be inferred from this in regard to the measure of an angle formed by two secants intersecting without the circumference?

Statement:—

153. Draw a circumference with any radius.

Take any external point and mark it P.

From P draw the tangents PM and PN.

Draw the chord MN.

Produce PM through M to any point A.

The angle AMN is equal to the sum of what two angles?

(See §77.)

Then the angle P is equal to the difference of what two angles?

By what is the angle AMN measured? (See §151.)

By what is the angle MNP measured?

Since the angle P is equal to the difference of the angles AMN and MNP, by what is it measured?

What may be inferred from this in regard to the measure of an angle formed by two intersecting tangents?

Statement :—

154. Draw a circumference with any radius.

From any external point P draw a tangent PC and a secant intersecting the circumference in the points A and B, respectively.

Draw the chord AC.

The angle BAC is equal to the sum of what two angles?

Then the angle P is equal to the difference of what two angles?

By what is the angle BAC measured? (See §135.)

By what is the angle ACP measured?

Since the angle P is equal to the difference of these two angles, by what is it measured?

What may be inferred from this in regard to the measure of an angle formed by a tangent and a secant?

Statement:—

155. Problem:—*To draw a circumference through three points not in the same straight line.*

Take any three points on your paper not in the same straight line and mark them A, B, and C.

Draw the straight lines AB and BC.

Draw the perpendicular bisectors of these two lines and mark their point of intersection O.

With O as a centre and a radius equal to OA draw a circumference.

If your construction is accurate the circumference will pass through the three points A, B, and C.

This method is used to circumscribe a circle about a triangle.

156.

EXERCISES.

1. How many degrees are there in an angle formed by two secants if the intercepted arcs are $\frac{1}{3}$ and $\frac{1}{10}$ of the circumference?

2. An angle formed by two secants contains 30° , and one of the intercepted arcs is twice the other; how many degrees are there in each arc?

3. How many degrees are there in the angle formed by two tangents, if the points of contact divide the circumference into two arcs one of which is twice the other.

4. An angle formed by a tangent and a secant intercepts

two arcs of which one contains 90° and the other 60° ; how many degrees are there in the angle?

5. Draw any triangle and circumscribe a circle about it. (See §155.)

157. It has been found that the circumference of any circle is a little more than three times the length of its diameter. If we could accurately measure the circumference and diameter of any circle and then divide the circumference by the diameter, the quotient to six places of decimals would be **3.141592 +**.

This result is very nearly equal to $\frac{22}{7}$, and in all calculations in this book we shall use this fraction and take the following equation to express the relation between the circumference and the diameter:

$$\text{Circumference} = \frac{22}{7} \times \text{diameter}.$$

Since the diameter is equal to twice the radius (**R**) we may substitute **2R** for the diameter in the above equation and we have

$$\text{Circumference} = 2 \times \frac{22}{7} \times R$$

Note:—The teacher should have two wooden cylinders, one 7cm. in diameter and the other 1dm. in diameter, to illustrate the relation between the circumference and diameter. Each pupil should measure the circumference and diameter of each cylinder and divide the circumference by the diameter. The circumference can be measured by rolling the cylinder and measuring the distance passed over in one revolution.

158.

EXERCISES.

1. What is the circumference of a circle whose diameter is 7 in. ? 21 in. ?
2. The radius of a circle is 3cm. 5mm. ; what is its circumference ?
3. What is the circumference of a circle whose radius is 14m. ? -
4. What is the radius of a circle whose circumference is 66 ft. ? 6m. 6dm. ? 2km. 2hm. ?

5. Draw a circumference with centre O and radius 1 in. Draw two radii OA and OB perpendicular to each other. What is the length of the circumference of this circle? What is the length of the arc AB?

6. Draw a circumference with centre O and radius of 3cm. 5mm.

Draw two radii OA and OB making an angle of 60° with each other.

What part of the circumference is the arc AB?

What is its length?

7. If the diameter of a circle is 1dkm. 4m., what is the length in meters of an arc of 45° ? Of 30° ? Of 40° ? Of 36° ? Of 270° ?

8. Draw a square with its sides 7cm. long.

Upon each side of the square as a diameter draw a semi-circumference outside the square.

What is the perimeter of the figure enclosed by these four semi-circumferences?

9. Again draw a square with its sides 7cm. long.

With the vertices as centres and a radius equal to 3cm. 5mm. draw four arcs outside the square and terminating at the middle points of the sides of the square.

What is the perimeter of the figure enclosed by these four arcs?

10. If the radius of the earth is assumed to be 3,960 mi., find the length of one degree on the circumference.

11. Draw an equilateral triangle with its sides each 6cm. 6mm. long.

Upon each side as a diameter draw a semi-circumference.

What is the perimeter of the figure enclosed by these three semi-circumferences?

12. Again draw an equilateral triangle with its sides each 6cm. 6mm. long.

With the vertices of the triangle as centres and a radius equal to 3cm. 3mm. draw three arcs outside the triangle and terminated by the sides of the triangle.

What is the perimeter of the figure enclosed by these three arcs?

13. What is the circumference of a circle whose radius is $7x$ in.?

14. What is the radius of a circle if the circumference is $44y$ ft.?

159. Problem:—*To divide the circumference of a circle into six equal parts.*

Draw a circumference with centre O and any radius.

Draw a chord AB equal in length to the radius. (See §129.)

Draw the radii OA and OB.

What kind of a triangle is OAB?

Is it equiangular? (See §75.)

How many degrees are there in the sum of its three angles?
(See §78.)

Then how many degrees are there in the angle AOB?

How many degrees are there in the arc AB?

What part of the circumference is the arc AB?

Now with B as a centre and a radius equal to the radius of the circle draw an arc cutting the circumference at C, then with C as a centre and the same radius cut the circumference at D, etc., etc.

The points A, B, C, D, etc., will divide the circumference into six equal parts.

160.

EXERCISES.

1. Draw a circumference with centre O and radius equal to 3cm.

Divide the circumference into six equal parts.

Letter the points of division A, B, C, etc.

Draw the chords AC, CE, and EA.

Are these chords equal?

What kind of a triangle is ACE?

Draw tangents at the points A, E, and C.

Mark the points where they intersect M, N, and P.

What kind of a triangle is MNP?

Draw an *altitude* of the triangle MNP and measure it.

The altitude of the circumscribed equilateral triangle MNP is how many times the radius of the circle?

Measure the altitude of the incircbed triangle ACE.

The altitude of the inscribed equilateral triangle is how many times the radius of the circle?

How do the altitudes of these triangles compare?

2. The radius of a certain circle is $1\frac{1}{2}$ in.; what is the altitude of the circumscribed equilateral triangle? Of the inscribed equilateral triangle?

3. If the altitude of an equilateral triangle is 9 ft. 6 in., what is the radius of the circle inscribed in it?

4. The diameter of a circle is 54m.; what are the altitudes of the inscribed and circumscribed equilateral triangles?

CHAPTER V.

POLYGONS.

161. Fig. 26 represents a polygon.

A polygon may have any number of sides.

Write a definition of a **polygon** similar to the definition of a triangle given in §68.

Definition :— .

The bounding lines are the **sides** of the polygon, the angles formed by the sides are the **angles** of the polygon, and the vertices of the angles are the **vertices** of the polygon.

A polygon of *three* sides is called a **triangle**.

A polygon of *four* sides is called a **quadrilateral**.

A polygon of *five* sides is called a **pentagon**.

A polygon of *six* sides is called a **hexagon**.

A polygon of *seven* sides is called a **heptagon**.

A polygon of *eight* sides is called an **octagon**.

A polygon of *ten* sides is called a **decagon**.

A polygon of *twelve* sides is called a **dodecagon**.

162. In Fig. 26 we have the polygon ABCDEF.

Two angles which have their vertices at the extremities of the same side are called *adjacent angles of the polygon*.

Name the angles in Fig. 26 which are adjacent to the angle A.

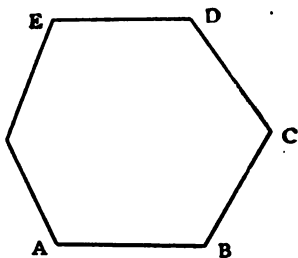


FIG. 26.

163. Draw a straight line in Fig. 26, connecting two vertices not adjacent, such as A and C. This line is a *diagonal* of the polygon.

Write a definition of a **diagonal** of a polygon.

Definition:—

164. Draw any pentagon and mark its vertices A, B, C, etc. Draw the diagonals AC and AD.

Into how many triangles do these diagonals divide the pentagon?

What is the difference between the *number of sides* of the pentagon and the *number of triangles*?

Draw any hexagon and mark its vertices A, B, C, etc.

Draw the diagonals AC, AD, and AE.

Is the difference between the number of sides and the number of triangles of the hexagon the same as we found in the case of the pentagon?

Can you explain why there is this difference between the number of sides and the number of triangles?

How many triangles would be formed by drawing the diagonals from one vertex of a *heptagon*? Of an *octagon*? Of a *decagon*? Of a *dodecagon*?

How many triangles would be formed by drawing the diagonals from one vertex of a polygon of n sides?

165. Draw any hexagon.

Mark the vertices A, B, C, etc.

Draw the diagonals from the vertex A.

Into how many triangles do these diagonals divide the hexagon?

What is the sum of the angles of each one of these triangles? (See §78.)

What is the sum of the angles of all the triangles?

Is the sum of the angles of the triangles equal to the sum of the angles of the polygon?

Then what is the sum of the angles of the polygon?

If the polygon had n sides, how many triangles would be formed by diagonals drawn from the vertex A ?

What would be the sum of the angles of each one of these triangles?

Then what is the sum of the interior angles of a polygon of n sides?

Statement.:—

166.

EXERCISES.

1. What is the sum of the interior angles of a pentagon? Of a hexagon? Of a heptagon? Of an octagon? Of a decagon? Of a dodecagon?

2. The sum of the interior angles of a polygon of n sides is equal to 20 right angles; what is the value of n ?

3. If the sum of the interior angles of a polygon is equal to 10 right angles, how many sides has the polygon?

4. How many sides has a polygon, if the sum of its interior angles is equal to 2 right angles? What name is given to a polygon of this number of sides?

5. What is the name given to a polygon, if the sum of its interior angles is equal to 4 right angles?

6. How many sides has a polygon, if the sum of its interior angles is equal to 540° ? 900° ? 360° ? 180° ?

167. Draw any polygon and mark its vertices A, B, C, D, etc.

Form the exterior angles of the polygon by producing its sides AB, BC, CD, etc., in succession.

What is the sum of the interior and exterior angle at each vertex? (See §48.)

If the polygon has n sides, what is the sum of the *exterior and interior angles*?

Statement :—

168.

EXERCISES.

1. What is the sum of the exterior and interior angles of a triangle? Of a quadrilateral? Of a pentagon? Of a hexagon? Of a heptagon? Of an octagon? Of a decagon? Of a dodecagon?

2. The sum of the exterior and interior angles of a certain polygon is equal to 10 right angles; what is the number of its sides?

3. The sum of the exterior and interior angles of a polygon is equal to 6 right angles; what is the number of its sides?

4. What is the number of sides of a polygon, if the sum of its exterior and interior angles is equal to 8 right angles?

5. What is the number of sides of a polygon, if the sum of its exterior and interior angles is equal to 540° ? 720° ? 1080° ?

6. What is the sum of the exterior and interior angles of a triangle?

What is the sum of its interior angles?

By subtracting the sum of its interior angles from the sum of its exterior and interior angles, find the sum of its exterior angles.

7. By the same method as in the preceding exercise find the sum of the exterior angles of a quadrilateral. Of a pentagon. Of a hexagon. Of a heptagon. Of an octagon. Of a decagon.

169. By the same method as used in exercise 7 of the preceding section find the expression for the sum of the exterior angles of a polygon of n sides. (See §165 and §167.)

What is the sum of the exterior angles of any polygon?

Statement:—

170.

EXERCISES.

1. A polygon has n sides and the sum of its interior angles is equal to the sum of its exterior angles; what is the value of n ?

2. The sum of the interior angles of a polygon is twice the sum of its exterior angles; what is the number of its sides?

3. What is the number of sides of a polygon, if the sum of its interior angles is ten times the sum of its exterior angles?

171. Definition:—A **regular polygon** is a polygon which is *equilateral and equiangular*.

We are already familiar with the equilateral triangle and the square, both of which are regular polygons.

All regular polygons may be inscribed in a circle.

172. Problem:—*To inscribe a square in a circle.*

Draw a circumference with any radius.

Draw two diameters perpendicular to each other.

Mark the extremities of the diameters A, B, C, and D in order around the circle.

Draw the chords AB, BC, CD, and DA.

How do we know that the figure ABCD is an inscribed square? (See §114 and §136.)

173. Problem:—*To inscribe a regular hexagon in a circle.*

Draw a circumference with any radius.

Divide the circumference into six equal parts by the method of §159.

Mark the points of division A, B, C, D, etc.

Draw the chords AB, BC, CD, etc.

Are these chords all equal by construction?

Each angle of the polygon is measured by what fractional part of the circumference? (See §135.)

Then is this hexagon regular?

Give the reason for your answer.

174. Draw a circumference with centre O and radius 1 in. long.

Inscribe a regular hexagon in this circle.

Mark the vertices of this hexagon A, B, C, etc.

Draw the radii OA, OB, OC, etc.

Definition:—The **centre** of a regular polygon is the centre of its circumscribed circle.

175. Definition:—The **radius** of a regular polygon is the distance from its centre to any vertex.

The radius of a regular polygon is equal to the radius of the circumscribed circle.

The lines OA, OB, OC, etc., in the figure you have just drawn are the radii of the polygon.

What is the length of the radius of this polygon?

176. Definition:—The angles formed by the successive radii of a regular polygon are called the **angles at the centre**.

Name the angles at the centre of the polygon just drawn.

How many degrees are there in each of them?

Explain how you obtained your answer.

177. Definition:—The **perimeter** of a polygon is the *sum of its sides*.

What is the perimeter of the hexagon just drawn?

178. Problem:—*To inscribe a regular octagon in a circle.*

Draw a circumference with any radius.

Draw two diameters perpendicular to each other.

Draw two other diameters so as to bisect the angles formed by the first two.

Mark the extremities of the diameters A, B, C., etc., in order around the circle.

Draw the chords AB, BC, CD, etc.

The figure ABCDEFGH is an *inscribed regular octagon*.

179.

EXERCISES.

1. The perimeter of a regular hexagon is how many times the radius?

2. What is the perimeter of a regular hexagon inscribed in a circle whose radius is 6 ft. 2 in? 5km. 5hm.? x ft? $(x+y)$ yd.?

3. How many degrees are there in each interior angle of a regular pentagon? Of a regular hexagon? Of a regular octagon? Of a regular decagon? (See §165.)

4. How many degrees are there in each of the exterior angles of the above mentioned polygons?

5. Draw a regular hexagon with a radius of 3cm.

Mark the centre O and the vertices A, B, C, etc.

Draw the six radii.

How does the number of angles at the centre compare with the number of sides?

What is the sum of all the angles at the centre? (See §41.)

Then how many degrees are there in each angle at the centre of a regular hexagon?

6. How many degrees are there in each angle at the centre

of a regular pentagon? Octagon? Decagon? Dodecagon? (See §41.)

7. What expression represents the number of degrees in each *angle at the centre* of a regular polygon of n sides?

8. What expression represents the number of degrees in each of the *interior angles* of a regular polygon of n sides?

9. How does the sum of the angles at the centre of a regular polygon compare with the sum of its exterior angles? (See §169.)

10. What is the perimeter of a regular octagon, if the length of one side is 1m. 5dm.?

11. How many sides has a regular polygon, if each angle at the centre contains 90° ? 60° ? 45° ?

12. Inscribe a regular octagon in a circle whose radius is $\frac{3}{4}$ in.

13. Draw a circumference with a radius of $\frac{3}{4}$ in. and divide it into six equal parts.

Letter the points of division A, B, C, etc., in succession.

What kind of a figure is formed by drawing the lines AC, CE, and EA?

Give the reason for your answer.

14. Inscribe a regular hexagon in a circle whose radius is 2 cm. and mark its vertices A, B, C, etc.

Draw all nine diagonals of the hexagon.

Mark the centre of the polygon O and beginning with the intersection of OA with FB mark the intersections of the diagonals $m, n, p, q, r, s, t, u, v, w, x$, and y .

Name two regular hexagons in the figure.

Name fourteen equilateral triangles, nine rectangles, six rhombi, thirty-six right triangles, and six isosceles triangles.

Shade the figure in such a way as to bring out a six pointed star.

15. Draw a circumference with a radius of $\frac{3}{4}$ in. and inscribe a regular hexagon in it.

Upon each side of this hexagon construct a regular hexagon. This is easily done by first drawing their circumscribing circles.

Now draw all the diameters of the seven regular hexagons.

By shading certain triangles form a tile pattern composed of equilateral triangles.

16. Make the same construction as in the preceding exercise and form a tile pattern composed of equilateral triangles and hexagons.

17. Make the same construction as in exercise 15 and form a tile pattern composed of rhombi.

18. Inscribe a regular octagon in a circle of which the radius is 2 cm.

Upon four alternate sides of this octagon construct squares.

Upon the remaining four sides complete regular octagons.

Shade the squares forming a tile pattern composed of squares and octagons.

19. Construct a portion of a checker-board.

20. Construct a tile pattern composed of regular dodecagons and triangles.

21. Is it possible to use other regular polygons in tiling? Explain how you obtain your answer.

180. A regular polygon of any number of sides may be inscribed in a circle by drawing radii of the circle making angles with each other equal to *the angles at the centre* of the required polygon and then drawing chords connecting the extremities of the successive radii.

181.

EXERCISES.

1. By the method mentioned in §180 inscribe a regular pentagon in a circle of 4cm. radius.

2. By the same method inscribe a regular decagon in a circle whose radius is $1\frac{1}{4}$ in.

CHAPTER VI.

AREAS OF PLANE FIGURES.

182. All plane figures have been defined as portions of a plane bounded by lines, either straight or curved. That portion of the plane surface contained within its bounding lines is called the *area* of the figure. Just as we have certain units for measuring and comparing the lengths of lines and magnitudes of angles, so we have units for measuring and comparing the areas of figures. It has been found convenient to take for a unit of area the area of a square having each of its sides equal to a unit of long measure. Thus a square having each of its sides one inch long is taken as a unit of area and is called a *square inch*. The area of a square having each of its sides one yard long is a unit of area called a *square yard*. A square with each of its sides one meter long is a unit of area called a *square meter*, etc., etc. If the area of a given figure is equivalent to ten times the area of a square whose sides are one foot long, the area of the figure is equal to ten square feet. If it were possible, we might find the area of any plane figure by dividing it into small squares having their sides equal to some unit of long measure and then, by counting the number of squares contained in the figure, we should have the measure of its area. Since all plane figures cannot be readily divided into squares, it is necessary to find other methods for determining the amount of area contained in them. It can be shown that the areas of all plane geometric figures may be found in terms of a given unit of area by measuring the lengths of certain lines in the figure in terms of the corresponding unit of length, and then taking the product (or a fraction of the product) of these lengths. In general when we speak of the *area of a figure* we mean the number of units of square measure contained in it.

In the following sections we shall deduce the methods of obtaining the areas of certain plane figures, beginning with a rectangle.

THE AREA OF A RECTANGLE.

183. Draw a rectangle ABCD with its base AB 8cm. long and its altitude AD 5cm. long.

Divide the base AB into centimeters.

From the points of division draw perpendiculars to AB terminating in DC.

(These lines divide the rectangle into *columns*.)

Divide the altitude AD into centimeters.

From the points of division draw lines parallel to AB terminating in BC.

(These lines divide the rectangle into *rows*.)

Is the rectangle now divided into small squares?

What *unit of area* is each one of these squares?

How many *columns* of squares are there?

How many *squares* are there in a *column*?

Then how many squares are there in the rectangle?

Since each of these squares is a *square centimeter*, what is the *area* of the rectangle in square centimeters?

Is the length of the base in centimeters the same as the number of columns?

Is the length of the altitude in centimeters the same as the number of squares in a column?

Then, if you multiply the length of the base in centimeters by the length of the altitude in centimeters, is the product equal to the area in square centimeters?

What general principle may be inferred from this in regard to the *area of any rectangle*?

Statement :—

184. How many square inches are there in one square foot?

How many square feet are there in one square yard?

How many square yards are there in one square rod?

Insert the proper numbers to make the following table complete.

	square inches (sq. in.)	equal 1 square foot.	(sq. ft.)
	square feet	" 1 "	yard. (sq. yd.)
	square yards	" 1 "	rod. (sq. rd.)
160	square rods	" 1 acre.	(A.)
640	acres	" 1 square mile.	(sq. mi.)

How many millimeters are there in one square centimeter?

Insert the proper numbers to make the following table complete.

	square millimeters (sq.mm.)	equal 1 square centimeter.	(sq. cm.)
"	centimeters	" 1 "	decimeter. (sq. dm.)
"	decimeters	" 1 square meter.	(sq. m.)
"	meters	" 1 "	dekameter. (sq. dkm.)
"	dekameters	" 1 "	hektometer. (sq. hm.)
"	hektometers	" 1 "	kilometer. (sq. km.)

185.

EXERCISES.

1. Draw a rectangle ABCD with its base AB 7cm. long and its altitude AD 4cm. 5mm. long.

Divide the base into centimeters.

From the points of division draw perpendiculars to AB terminating in DC.

Divide the altitude AD into centimeters.

From the points of division draw lines parallel to the base, terminating in BC.

Into how many square centimeters and fractions of a square centimeter is the rectangle divided?

Is the area of this rectangle equal to the product of its base and altitude?

2. If A stands for the area, a for the altitude, and b for the

base of a rectangle, what equation can you write which gives the relation between these quantities?

3. The base of a rectangle is 9 ft. and its altitude is 7 ft.; what is its area?

4. The base of a rectangle is 7 ft. 6 in., and its altitude is 5 ft.; what is its area in square feet?

5. The base of a rectangle is 10cm. 5mm., and the altitude is 6cm. 7mm.; what is the area in square centimeters? In square millimeters?

6. Find the area of a rectangle in square decimeters, if the base is 25cm. 9mm. and the altitude 11cm. 3mm.

7. The area of a rectangle is 150 sq. ft. and the base is 15 ft.; what is the altitude?

8. The area of a rectangle is 200 sq. m. and the altitude is 2dkm.; what is the length of the base in meters?

9. The area of a rectangle is 128 sq. m. and the base is twice the altitude; find the length of the base and of the altitude.

10. If the base of a rectangle is three times the altitude and the area is 3 sq. m. 89 sq. dm. 80 sq. cm., what is the length of the base and altitude in decimeters?

11. A rectangular grass plot is 90 ft. long and 50 ft. wide. Around the outside of this grass plot is a walk 10 ft. wide. How many square feet are there in the walk?

12. The sum of the base and altitude of a rectangle is 75 and the base is 4 times the altitude; what is the area of the rectangle?

13. What is the area in square inches of the paper on which you are working?

14. What is the area of a square, if the length of each side is 6 in.?

15. The perimeter of a square is 20 in.; what is its area?

16. A flower garden is in the form of a square. The distance around it is 40 yds., and there is a walk 4 ft. wide all

around the garden; what is the area of the garden? Of the walk?

17. The area of a square is 20 sq. m. 25 sq. dm.; what is the length of its side in decimeters?

18. A square field contains $15\frac{5}{8}$ acres; find the length in rods of the fence which encloses it.

19. What will it cost to gravel a walk 1 yd. wide around the inside of a square court whose side is 127 yds., at 12c. per square yard?

20. A chain 80 ft. long encloses a rectangle 15 ft. wide; how much more surface would it enclose if the figure were a square?

21. A rectangle 25m. long is equivalent to a square whose side is 15m.; which has the greater perimeter?

22. Find the length and width of a rectangle whose area is 216 sq. ft. and perimeter 60 ft.

23. A rectangle is 36 ft. by 20 ft. If the length is diminished by 6 ft., by how much must the width be increased in order that the area may remain the same?

186. Draw a triangle ABC having the base AB 3 in. long, the side BC $2\frac{1}{2}$ in. long, and the included angle B a right angle.

Construct squares on each of the three sides of the triangle.

What is the area of the square constructed on the hypotenuse?

What is the area of each of the squares constructed on the legs?

What is the sum of the areas of the squares constructed on the legs?

How does this sum compare with the area of the square constructed on the hypotenuse?

What general principle may be inferred from this in regard to the relation of the areas of the squares constructed on the hypotenuse and legs of a right triangle?

Statement:—

187.

EXERCISES.

1. The two legs of a right triangle are 6 in. and 9 in., respectively; what is the area of the square constructed on the hypotenuse?

2. If the hypotenuse of a right triangle is 5 in. and one leg is 4 in., what is the length of the other leg?

3. The lengths of the three sides of a right triangle are $3x$, $4x$, and $5x$, respectively, and the area of the square on the hypotenuse is 2500 sq. ft.; what are the lengths of the three sides in feet?

4. If the hypotenuse of a right triangle is twice the short leg and the area of the square on the other leg is 48 sq. yds., what is the length of the short leg and of the hypotenuse?

5. Find the area of a rectangle whose width is 75m. and whose diagonal is 125m. long.

6. What is the length of the diagonal of a square whose side is 1 in. long?

7. The distance from the centre of a circle to a chord 10 ft. long is 12 feet; find the radius of the circle.

8. The radius of a circle is 6 in.; find the lengths of the tangents drawn from a point 10 in. from the centre.

9. From the end of a tangent 20 in. long a secant is drawn through the centre of the circle. If the external segment of the secant is 8 in., what is the radius of the circle?

10. The radius of a circle is 9 in. and the length of a tangent is 12 in.; find the length of a line drawn from the end of the tangent to the centre of the circle.

11. Two circles have radii of 7 in. and 1 in., respectively, and the distance between their centres is 10 in.; find the length of their common external tangents.

12. Find the length of their common internal tangents.

THE AREA OF A PARALLELOGRAM.

188. Draw on a separate piece of paper a parallelogram ABCD with the base AB 9cm. long, the side AD 8cm. long, and the included angle A equal to 30° .

(Place the letters inside the figure.)

From B draw a line perpendicular to AD, meeting AD at E.

This line BE is the altitude of the figure. What is its length?

Cut out the parallelogram with the scissors.

Now cut off the triangle BCE along the line BE.

Place the two parts of the parallelogram together on the desk in their original relative position.

Is the area of the parallelogram equal to the area of the triangle and the area of the trapezoid added together?

Move the triangle to the other end of the trapezoid so that BC of the triangle and AD of the trapezoid coincide.

What kind of a figure do the two parts now form?

Is the area of the rectangle equal to the area of the triangle and the area of the trapezoid added together?

Then is the area of the rectangle equal to the area of the original parallelogram?

Give the reason for your answer.

The area of the rectangle is equal to the product of what two quantities?

Have the rectangle and parallelogram the same base?

Have they the same altitude?

Then the area of the parallelogram is equal to the product of what two quantities?

What general principle may be inferred from this in regard to the area of any parallelogram?

Statement:—

189.**EXERCISES.**

1. If **A** stands for the area, **a** for the altitude and **b** for the base of a parallelogram, write the equation which expresses the relation between these quantities.

2. Construct a rhomboid ABCD with the base AB $2\frac{1}{2}$ in. long, the side AD $1\frac{1}{2}$ in. long, and the included angle A equal to 53° .

What is the altitude of this rhomboid?

What is its area?

3. Construct a rhomboid ABCD with the base AB 5cm. 5mm. long, the altitude 3cm. 4mm. long, and the angle DAB equal to 45° .

What is the area of this rhomboid?

4. How many different rhomboids can you construct with a base of 3 in. and an altitude of 2 in.?

5. The area of a parallelogram is 126 sq. m. and its altitude is 6m.; what is the length of its base?

6. The area of a certain rhomboid is 150 sq. m. 50 sq. dm. and its base is 20 m.; what is its altitude?

7. The base of a parallelogram is 7m. 9dm. and the altitude is 9m. 7dm.; what is its area in square meters? In square decimeters?

8. The base of a certain parallelogram is x in. and the altitude is y in.; what is the area in terms of x and y ?

190. Draw a rhombus ABCD with its sides 5cm. long and the angle A equal to 74° .

Draw the diagonals AC and BD.

What is the altitude of this rhombus?

What is its area?

Measure the lengths of the diagonals.

What is the product of the diagonals?

What is the relation between the area of the rhombus and the product of its diagonals?

What may be inferred from this in regard to *the area of any rhombus?*

Statement :—

191.**EXERCISES.**

1. If **A** stands for the area of a rhombus and **x** and **y** for the diagonals, write the equation which expresses the relation between these quantities.

2. Draw a rhombus ABCD with its sides $2\frac{1}{2}$ in. long and the angle A equal to 53° .

Find the area of this rhombus, taking the product of its base and altitude.

Find the area, taking half the product of its diagonals.

How do your results compare?

3. The diagonals of a rhombus are 12 ft. and 20 ft., respectively; what is its area?

4. The sum of the diagonals of a rhombus is 27m. and one diagonal is twice as long as the other; what is the area of the rhombus?

5. The length of one diagonal of a rhombus is x feet and of the other y feet; what is the area, if $x=10$, and $y=6\frac{3}{4}$?

6. The diagonals of a diamond-shaped pane of glass are 48cm. and 64cm.; find the number of square meters in 600 such panes.

192. THE AREA OF A TRIANGLE.

Draw a triangle ABC with the base AB $3\frac{1}{8}$ in. long, the side AC $1\frac{7}{8}$ in. long, and the side BC $2\frac{1}{2}$ in. long.

Draw the altitude from the vertex C.

Measure the altitude and mark its length on the figure.

From the vertex C draw a line parallel to AB.

From the vertex B draw a line parallel to AC.

Mark the point where these two lines intersect D.

The area of the parallelogram ABCD is equal to the product of what two quantities?

Has the triangle ABC the same base as the parallelogram ABCD?

Has it the same altitude?

What part of the parallelogram is the triangle? (See §121.)

Give the reason for your answer.

Then the area of the triangle ABC is equal to what?

What general principle may be inferred from this in regard to the area of any triangle?

Statement:—

193. EXERCISES.

1. If **A** stands for the area, **a** for the altitude, and **b** for the base of a triangle, what equation can you write which expresses the relation between these three quantities?

2. The base of a triangle is 6m. 2dm. 5cm. and the altitude is 7m. 2dm.; what is its area?

3. The base of a certain triangle is twice as long as its altitude and its area is 5 sq. m. 10 sq. dm. 76 sq. cm.; what is the length of its base and of its altitude?

4. The base and altitude of a triangle are equal to each other and their sum is 32 yds.; what is the area of the triangle?

5. The base of a triangle is 15 ft. and its altitude is 8 ft.; find the perimeter of an equivalent rhombus whose altitude is 6 ft.

6. The area of a triangle is 700 sq. in. and the altitude is 35 in.; what is the length of the base in feet?

7. The area of a certain triangle whose altitude is 20cm. and whose base is 40cm. is four times the area of a certain square; what is the length of a side of the square?

8. Construct a triangle ABC with the base AB $1\frac{3}{4}$ in., the side AC $1\frac{1}{2}$ in., and the included angle A equal to 53° .

What is the area of this triangle?

9. The altitude of a triangle is half its base and its area is 81 sq. yd.; what is the length of its base and of its altitude?

10. If the base of a triangle is x feet and its altitude is y feet, what is its area?

11. The sum of the base and altitude of a triangle is 36 rd. and the base is three times the altitude; what is the area of the triangle in square rods?

12. Construct a triangle ABC with the base AB $2\frac{1}{4}$ in., the side AC $1\frac{1}{2}$ in., and the included angle A equal to 90° .

What kind of a triangle is this?

What is its area?

13. Find the area in square centimeters of a right triangle whose legs are 4cm. 5mm. and 3cm. 2mm.

14. Find the area in square meters of a right triangle whose legs are 9m. 2dm. 3cm. 5mm. and 7m. 1dm. 4cm. 2mm.

15. The area of a right triangle is 225 sq. km. and the length of one leg is 15 km.; what is the length of the other leg?

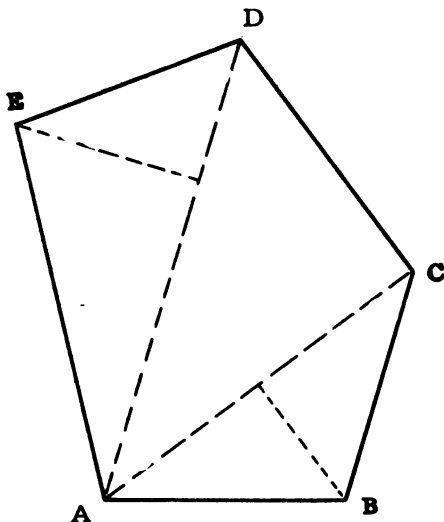
16. The area of a right triangle is 49 sq. m. 35 sq. dm. and one leg is 5m. 2dm. 5cm.; what is the length of the other leg in meters and decimeters?

17. The hypotenuse of a right triangle is 50.75 ft., and the area is 101.5 sq. ft.; what is the length of the perpendicular drawn from the vertex of the right angle to the hypotenuse?

18. The legs of a right triangle are 15 ft. and 20 ft.; what is the length of the perpendicular drawn from the vertex of the right angle to the hypotenuse?

19. In a right triangle the hypotenuse is 13 ft. and one leg is 5 ft.; find the area.

20. Find the area, in square inches, of the polygon given below by finding the areas of the three triangles and adding them.



AREA OF A TRAPEZOID.

194. Draw a trapezoid ABCD with the lower base AB 5cm., the upper base DC 4cm., and the altitude 3cm.

Draw the diagonal AC.

Is the altitude of the triangle ABC the same as the altitude of the trapezoid?

Is the altitude of the triangle CDA the same as the altitude of the trapezoid?

Is the area of the trapezoid equal to the sum of the areas of the two triangles?

What is the area of the triangle ABC?

What is the area of the triangle CDA?

Then what is the area of the trapezoid?

Multiply the altitude of the trapezoid by the sum of its bases.

The area of the trapezoid is what fraction of this product?

What may be inferred from this in regard to *the area of any trapezoid?*

Statement:—

195.

EXERCISES.

1. If **A** stands for the area, **B** for the lower base, **b** for the upper base, and **a** for the altitude of the trapezoid, what equation can you write which expresses the relation between these quantities?

2. If for a particular trapezoid $B=25$, $b=15$, and $a=9$, what is its area?

3. The upper base of a trapezoid is 7m. 6dm. 4cm., the lower base is 3m. 3dm. 6cm., and the altitude is 1dkm. 2m. What is its area in square meters?

4. What is the area of a trapezoid whose altitude is 15 ft. and the sum of whose bases is 22 yds.?

5. The area of a trapezoid is 92 sq. m. and its altitude is 4m. 6dm.; what is the sum of its bases?

6. The area of a trapezoid is 1445 sq. ft., the upper base is 16 ft., and the lower base is 18 ft.; what is its altitude?

7. The area of a trapezoid is 160 sq. yds., the altitude is x ft., the upper base is $2x$ ft., and the lower base $3x$ ft.; what are the lengths of the bases and the altitude?

8. Draw any trapezoid and find its area.

9. Draw a triangle ABC having the sides AB, BC, and CA equal, respectively, to 6cm. 4mm., 5cm. 5mm., and 4cm. 8mm.

Mark a point D on AC 3cm. from A.

Through D draw a line parallel to AC, cutting BC at a point E.

Find the area of the trapezoid ABED. (Find the required lengths with the scale.)

10. The bases of a trapezoid are 8 ft. and 10 ft. and the altitude is 6 ft.; find the base of an equivalent rectangle which has the same altitude.

196. Draw a trapezoid ABCD having its lower base AB 6cm. 4mm., its upper base DC 4cm. 6mm., and its altitude 3cm. long.

Draw the median and mark it MN. (See §122.)

What is the area of this trapezoid?

What is the length of the median? (See §123.)

What is the product of the median by the altitude?

What may be inferred from this with regard to *the area of a trapezoid*?

Statement:—

197. EXERCISES.

1. If the median of a trapezoid is 8km. 6hm. and its altitude is 9km. 5hm., what is its area?

2. If the area of a trapezoid is 6 sq. m. 25 sq. dm. and the length of the median is 2m. 5dm. ; what is its altitude?
3. The area of a trapezoid is 6sq. m. 75sq. dm. and its altitude is 1m. 5dm. ; what is the sum of its bases?
4. If the length of the lower base of a trapezoid is x , the upper base y , and the altitude z , what is the area of the trapezoid in terms of x , y , and z ?
5. If the area of a trapezoid is $4xy$ and the length of the altitude is y , what is the length of the median?
6. The area of a trapezoid is 32 sq. ft., one base is 10 ft., and the altitude is 4 ft. ; what is the length of the other base?
7. Find the area of a trapezoid whose bases are 73 ft. and 57 ft., respectively, and each of whose legs is 17 ft.
8. In what two ways may the area of a trapezoid be found?

AREA OF A REGULAR POLYGON.

198. Inscribe a regular hexagon in a circle whose radius is 3cm. (See §173.)

Mark the centre of the circle O and the vertices of the polygon A, B, C, D , etc.

Draw the radii OA, OB, OC , etc.

Draw a perpendicular from O to the side AB and mark the point where it meets AB with the letter P .

The line OP is called the *apothem* of the polygon.

Write a definition of the **apothem** of a regular polygon.

Definition :—

199. Measure the length of the apothem of the hexagon you have just drawn and mark its length on the figure.

What is the area of the triangle OAB?

Are the other five triangles equal to the triangle OAB?

Is the area of the hexagon equal to the sum of the areas of the triangles?

What is the area of the hexagon?

200. Inscribe a regular octagon in a circle whose radius is 4cm. (See §178.)

Mark the centre of the circle O and the vertices of the octagon A, B, C, D, etc.

Draw the apothem to the side AB.

If we denote the length of a side of the octagon by s and the length of the apothem by a , to what is the area of the triangle OAB equal?

Give the reason for your answer.

How many triangles are there in the figure?

Then to what is the area of the polygon equal?

Is this the same as $\frac{1}{2} \times a \times 8s$?

What is the length of the *perimeter* of the octagon?

We may infer from this that *the area of any regular polygon is equal to one half the product of what two quantities?*

Statement:—

201.

EXERCISES.

1. Inscribe a regular octagon in a circle whose radius is 5cm. and find its area.

2. If P stands for the perimeter, a for the apothem, and A

for the area of a regular polygon, write the equation which expresses the relation between these three quantities.

3. The perimeter of a certain regular polygon is 9m. 4dm. 6cm. and its apothem is 4m. 5dm.; what is its area?

4. The area of a regular hexagon is 259.8 sq. ft. and the length of one side is 10 ft.; what is the length of the apothem?

5. The apothem of a regular polygon is half the length of one side and its area is 625 sq. m.; what is the length of the perimeter, if the polygon has four sides?

AREA OF A CIRCLE.

202. Draw a circumference with centre O and radius equal to $1\frac{3}{4}$ in.

Draw two radii OA and OB making an angle of 10° with each other.

Let **a** stand for the length of the arc AB and let **R** stand for the length of the radius.

If we consider the sector OAB as a triangle having **a** for its base; what is its altitude?

Then what is its area? (See §192.)

Into how many sectors equal to OAB can we divide the circle?

Then to what is the area of the circle equal?

Is this the same as $\frac{1}{2} \times R \times 36a$?

Is **36a** equal to the *circumference* of the circle?

Then *the area of a circle is equal to one half the product of what two quantities?*

Statement:—

203.

EXERCISES.

1. If **A** stands for the area, **R** for the radius, and **C** for the circumference of a circle; what equation can you write which gives the relation between these three quantities?

What is the relation between the circumference of a circle and its radius? (See §157.)

If we substitute for **C** its value in terms of **R** in the equation just written, what does the equation become?

2. What is the area of a circle whose radius is 14 in.? 21 in.? $12\frac{1}{2}$ in.? $3\frac{1}{2}$ in.?

3. What is the area of a circle whose diameter is 7m.?

4. What is the radius of a circle whose area is 154 sq. ft.? 13sq. m. 86sq. dm.?

5. Draw a circle with centre **O** and radius 5cm. 6mm.

Draw two radii **OA** and **OB** perpendicular to each other.

What part of the circle is the sector **OAB**?

What is the area of the circle?

What is the area of the sector?

6. Draw a circle with centre **O** and radius 5cm. 6mm.

Draw two radii **OA** and **OB**, making an angle of 60° with each other.

What part of the circle is the sector **AOB**?

What is the area of this sector?

7. If the radius of a circle is 63 in., what is the area of a sector whose angle at the centre is 30° ? 36° ? 40° ? 45° ? 180° ? 270° ?

8. Draw an equilateral triangle having its sides 7cm. long.

Upon each side as a diameter construct a semi-circumference.

What is the area of the trefoil bounded by the three semi-circumferences?

9. Again draw an equilateral triangle having its sides 7cm. long.

With the vertices as centres and a radius equal to 3cm. 5mm. draw three arcs outside the triangle and terminated by its sides.

What is the area of the trefoil in square centimeters?

10. Draw a square having its sides 6cm. 3mm. long.

Upon the sides as diameters construct semi-circumferences. Find the total area of the figure.

11. Draw a square having its sides 5cm. 6mm. long.

With the vertices as centres and a radius equal to 2cm. 8mm. draw arcs outside of the square and terminated by its sides.

Find the total area of the figure in square centimeters.

12. Find the area of a circular ring included between two concentric circumferences whose radii are 2cm. 8mm. and 3cm. 5mm., respectively.

13. Each of two fields contains 38 sq. m. 50 sq. dm. One is in the form of a square and the other of a circle. How many more meters of fencing will be required for one field than for the other?

14. Three circles, each having a radius of 35 in., touch each other. What is the area of the three-pointed figure thus formed?

CHAPTER VII.

RATIO, PROPORTION, AND SIMILAR FIGURES.

204. Definition:—The **ratio** of two quantities of the same kind is the *quotient* obtained by dividing the numerical measure of one by the numerical measure of the other, when their measures are both expressed in terms of the same unit.

Thus, if a line AB is 3 in. long and another line CD is 5 in. long, the ratio of AB to CD is $\frac{3}{5}$.

A ratio is sometimes written with a colon. Thus the ratio of 3 to 5 may be written 3 : 5, and is read "the ratio of 3 to 5."

The *first term* of a ratio is called the **antecedent** and the *second term* the **consequent**.

205.

EXERCISES.

1. What is the ratio of two lines whose lengths are 4 in. and $1\frac{1}{2}$ in.? 1 ft. 3 in. and 1 ft. 8 in.? 2 cm. 4mm. and 3 cm. 6mm.?

2. Draw a triangle ABC whose sides AB, BC, and CA are equal, respectively, to $1\frac{1}{2}$ in., $2\frac{1}{2}$ in., and $3\frac{1}{2}$ in.

What is the ratio of AB to BC? Of BC to CA? Of CA to AB?

3. What is the ratio of two angles which contain 30° and 45° , respectively? Which contain $5^\circ 15'$ and $6^\circ 45'$?

4. What is the ratio of two triangles whose areas are 1 sq. m. and 5 sq. cm., respectively?

5. What is the ratio of an angle of 20° to its complement?

6. What is the ratio of an angle of 80° to its supplement?

7. What is the ratio of an arc of 150° to a semi-circumference?

8. One angle contains 20° and another 30° ; what is the ratio of the first to the second? Of the second to the first? Of the sum of the two angles to the first? Of their difference to the second?

9. The side of one square is 4m. and of another 5m.; what is the ratio of the perimeters of the two squares?

10. The side of one regular hexagon is 2yds. and of another 7 yds.; what is the ratio of the perimeters of the two regular hexagons?

206. *Definition*.—A **proportion** is a statement of equality between two or more ratios.

Thus $\frac{3}{6} = \frac{4}{8}$ is a proportion.

This may be written $3:6=4:8$ or $3:6::4:8$, and is read “3 is to 6 as 4 is to 8.”

The four quantities in the proportion are called the **terms** of the proportion.

The *first* and *last terms* are the **extremes** and the *second* and *third*, the **means**.

If two or more ratios are equal, their terms are said to be **proportional**.

207. Having given the proportion $a:b=c:d$, prove the following statement:

Statement.—In every proportion the product of the means is equal to the product of the extremes.

208. EXERCISES.

Answer the following questions by using the principle stated in the preceding section.

1. If 2, 3, and 6 are the first three terms of a proportion, what is the fourth term?

2. Find the value of x in each of the following proportions:

(i) $x:7=9:21$.

(iv) $2\frac{1}{2}:2\frac{1}{4}=10:x$.

(ii) $8:x=12:15$.

(v) $x:18=\frac{2}{3}:\frac{4}{3}$.

(iii) $3:4=x:12$.

(vi) $\frac{3}{4}:6=\frac{4}{5}:x$.

209. When the *means* of a proportion are *equal*, either of them is called a **mean proportional** between the first and last terms.

The *fourth term* in such a case is called a **third proportional** to the other two.

Thus, in the proportion $4 : 6 = 6 : 9$, 6 is a *mean proportional* between 4 and 9, and 9 is a *third proportional* to 4 and 6.

210.**EXERCISES.**

1. Find a mean proportional between 2 and 8.

Solution :—

Let x be the mean proportional.

Then $2 : x = x : 8$

Therefore $x^2 = 16$

“ $x = 4$. Ans.

2. Find the mean proportional between 3 and 12. Between 4 and 16. Between 3 and 27. Between 5 and 20.

3. Find a third proportional to 2 and 4. To 3 and 9. To 4 and 12. To 5 and 10.

211. Draw a triangle ABC having its sides AB, BC, and CA equal, respectively, to 6cm., 5cm., and 7cm. 5mm.

Mark a point M on the side AC making AM equal to 3cm.

Through M draw a line parallel to AB, meeting BC at a point N.

What is the ratio of AM to MC?

What is the ratio of BN to NC?

Are these two ratios equal?

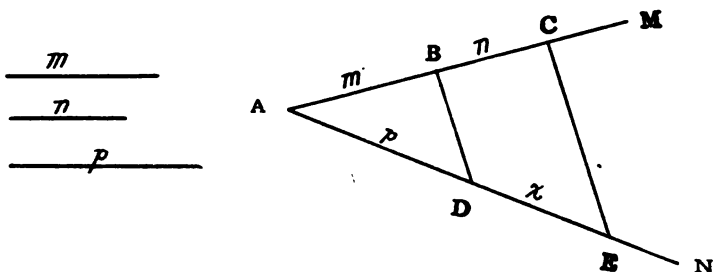
Remark: Two lines are divided *proportionally* when the ratio of the segments of one is equal to the ratio of the segments of the other.

Are the sides AC and BC divided proportionally by the line MN?

What may be inferred from this in regard to the division of two sides of a triangle by a line parallel to the third side?

Statement :—

212. Problem :—*To find a fourth proportional to three given straight lines.*



Let m , n , and p be the three given straight lines.

Draw any two lines such as AM and AN , making any angle with each other.

On AM lay off AB equal to m and BC equal to n .

On AN lay off AD equal to p .

Draw a line from B to D .

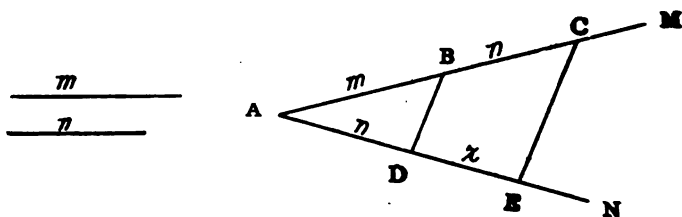
Through the point C draw a line parallel to BD , meeting AN at E .

Then DE is the fourth proportional to m , n , and p .

Measure the lengths of m , n , p , and DE and see whether they form a true proportion.

Explain how this construction depends upon the principle stated in §211.

213. Problem:—*To find a third proportional to two given straight lines.*



Let m and n be the two given straight lines.

Draw two lines such as AM and AN , making any angle with each other.

On AM lay off AB equal to m and BC equal to n .

On AN lay off AD equal to n .

Draw the line BD .

Through C draw a line parallel to BD , meeting AN at E .

Then DE is a third proportional to m and n .

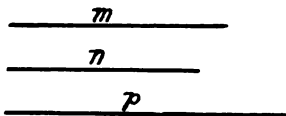
Explain how this construction depends upon the principle stated in §211.

Measure m , n , and DE and see whether they form a true proportion.

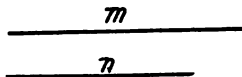
214.

EXERCISES.

- Find a fourth proportional to the three lines below.



- Find a third proportional to the two lines below.



215. Draw a triangle ABC having its sides AB, BC, and CA equal, respectively, to $2\frac{1}{2}$ in., 3 in., and 2 in.

Draw the bisector of the angle C and mark the point where it intersects the side AB with the letter D. (See §61.)

What is the ratio of AC to CB?

What is the ratio of AD to DB?

Are these two ratios equal?

Is the side AB divided into segments which are proportional to the other two sides?

What may be inferred from this in regard to the manner in which *the bisector of an angle of a triangle divides the opposite side*?

Statement :—

216. A line may be divided **internally** or **externally**.

A line is divided *internally* when the point of division lies *between the extremities of the line*.

The point M divides the line AB *internally* into the segments MA and MB.



A line is divided *externally* when the point of division lies *on the line produced*.

The point N divides the line AB *externally* into the segments NA and NB.

In either case the two segments of the line are the parts of the line included between the point of division and the extremities of the line.

217. Draw a triangle ABC having its sides AB, BC, and CA equal, respectively, to $1\frac{3}{4}$ in., 3 in., and $1\frac{1}{2}$ in.

Produce BC to D, forming the exterior angle ACD.

Draw the bisector of the exterior angle ACD and produce BA to meet it at M.

What is the ratio of MA to MB?

What is the ratio of CA to CB?

Does the point M divide the side AB externally into segments which are proportional to the other two sides?

What may be inferred from this in regard to the manner in which *the bisector of an exterior angle of a triangle divides the opposite side*?

Statement:—

218.

EXERCISES.

1. The sides AB, BC, and CA of a triangle ABC are 2 ft., 3 ft., and 4 ft., respectively.

What are the ratios of the segments of the sides formed by the respective bisectors of the angles?

2. The sides AB, BC, and CA of a triangle ABC are 1dm. 8cm., 2dm. 4cm., and 3dm. 6cm., respectively.

What is the ratio of the segments of the side CA formed by the bisector of the exterior angle at the vertex B?

219. Draw a triangle ABC having its sides AB, BC, and CA equal, respectively, to 10cm., 8cm., and 6cm.

What kind of an angle is C.

What kind of a triangle is this?

Drop a perpendicular from the vertex C to the side AB and mark its foot D.

What is the length of this perpendicular?

What are the lengths of the segments of the hypotenuse formed by the perpendicular?

Is the perpendicular a mean proportional between the segments of the hypotenuse?

What may we infer from this in regard to *the perpendicular dropped from the vertex of the right angle to the hypotenuse of a right triangle?*

Statement:—

220. Referring to the triangle of the preceding section, is the leg BC a mean proportional between the whole hypotenuse and the adjacent segment DB?

Is the leg AC a mean proportional between the whole hypotenuse and the adjacent segment AD?

What may we infer from this concerning any right triangle?

Statement:—

221. Draw a straight line AB $2\frac{1}{2}$ in. long.

Upon AB as a diameter draw a semi-circumference.

From any point C in the semi-circumference drop a perpendicular to the diameter AB and mark its foot D.

Draw AC and BC.

What kind of an angle is $\angle ACB$? (§136.)

Is CD a mean proportional between the segments of the diameter?

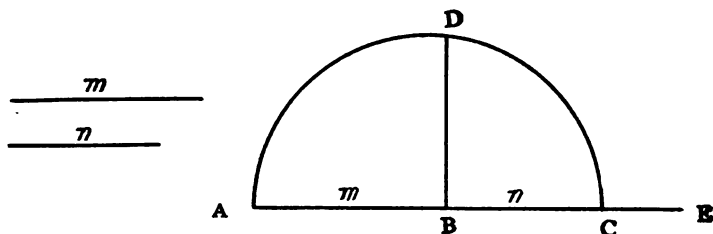
Give the reason for your answer.

What may be inferred from this in regard to *the perpendicular dropped from any point in a semi-circumference to the diameter*?

Statement:—

222. Problem:—*To find a mean proportional between two given straight lines.*

The figure of the preceding section suggests the method of construction.



Let m and n be the two given straight lines.

On any line such as AE lay off AB equal to m and BC equal to n .

Upon AC as a diameter construct a semi-circumference.

At the point B erect a perpendicular meeting the circumference at D .

Then BD is a mean proportional between m and n .

223.**EXERCISES.**

1. Find a mean proportional between the two lines below.



Measure the given lines and the mean proportional and see whether they form a true proportion.

2. The hypotenuse of a right triangle is 10 ft. and the legs are 8 ft. and 6 ft., respectively. What are the lengths of the segments of the hypotenuse formed by the perpendicular drawn from the vertex of the right angle to the hypotenuse?

What is the length of the perpendicular?

3. The legs of a right triangle are 9m. and 12m., respectively.

What is the area of the triangle?

What is the length of the hypotenuse?

What are the lengths of the segments of the hypotenuse formed by the perpendicular from the vertex of the right angle?

What is the length of the perpendicular?

224. *Similar polygons* are polygons which have the same shape.

The two polygons ABCDE and A'B'C'D'E' (Fig. 27) are similar.

Trace the polygon A'B'C'D'E' on thin paper and, by applying the tracing to the polygon ABCDE, see whether *the corresponding angles of the two polygons are equal*.

Measure the sides of both polygons in metric units.

What is the ratio of A'B' to AB? Of B'C' to BC? Of C'D' to CD? Of D'E' to DE?

Are *the corresponding sides* of the two polygons *proportional*? (See §206.)

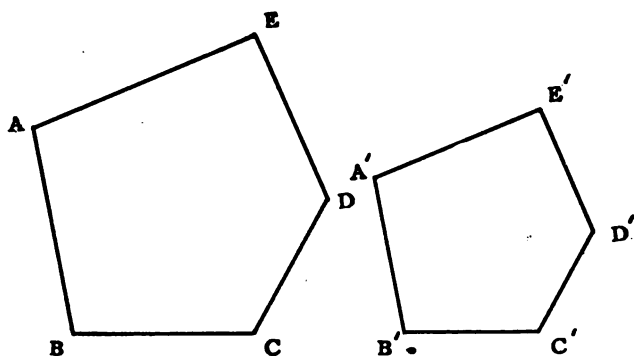


FIG. 27.

Write a definition of **similar polygons** which states the relation between the corresponding angles and the corresponding sides.

Definition :—

225. Draw a triangle ABC having its sides AB, BC, and CA equal, respectively, to 6cm. 4mm., 7cm. 2mm., and 8cm.

Draw another triangle A'B'C' having its sides A'B', B'C' and C'A' equal, respectively, to 4cm. 8mm., 5cm. 4mm., and 6cm.

See whether the two triangles are similar according to the definition of similar polygons written in the preceding section.

226.**EXERCISES.**

1. Draw a triangle ABC having its sides AB, BC, and CA equal, respectively, to 10cm., 8cm., and 6cm.

Drop a perpendicular from C to AB and mark its foot D.

Is the triangle ABC similar to the triangle ADC?

Is the triangle ABC similar to the triangle DBC?

2. The sides of a triangle are 6, 7, and 8. In a similar triangle the side corresponding to 8 is 40. Find the other two sides.

3. The shadow cast on level ground by a flagstaff is 30m. long; at the same time the shadow cast by a vertical rod 5 ft. high is 3 ft. long. Find the height of the flagstaff.

4. The legs of a right triangle are 10m. and 15m. and the shorter leg of a similar right triangle is 8m.; find the other leg.

5. The legs of a right triangle are 3 ft. and 4 ft. What is the hypotenuse of a similar right triangle whose shorter leg is 9ft.?

6. A tree 90 ft. high stands on the bank of a stream. At a certain hour it casts a shadow directly across the stream and just reaching the opposite bank. If at the same time a telegraph pole 40 ft. high casts a shadow 30 ft. long, what is the width of the stream?

7. Calculate the height of the school flagpole by means of its shadow at 1 o'clock p. m.

227. Draw a triangle ABC having its sides AB, BC, and CA equal, respectively, to 7cm. 2mm., 6cm. 8mm., and 5cm. 6mm.

Draw another triangle A'B'C' having its sides A'B', B'C', and C'A' equal, respectively, to three quarters of the sides of the triangle ABC.

Are these triangles similar? Give the reason for your answer.

What is the ratio of their corresponding sides?

What are the perimeters of these triangles?

Have the perimeters the same ratio as any two corresponding sides?

What may be inferred from this in regard to the ratio of the perimeters of any two similar triangles?

Statement :—

228. Draw a triangle ABC having its sides AB, BC, and CA equal, respectively, to $3\frac{1}{8}$ in., $2\frac{1}{2}$ in., and $1\frac{7}{8}$ in.

Draw another triangle A'B'C' having its sides A'B', B'C', and C'A' equal, respectively, to $\frac{1}{2}$ of the sides of the triangle ABC.

Are these two triangles similar? Give the reason for your answer.

Draw the altitude CD in the triangle ABC and the altitude C'D' in the triangle A'B'C'.

Measure these altitudes.

What is the ratio of C'D' to CD?

Is the ratio of these altitudes the same as the ratio of any two corresponding sides?

What may be inferred from this in regard to the ratio of corresponding altitudes of two similar triangles?

Statement :—

229.**EXERCISES.**

1. The bases of two similar triangles are 3 in. and 5 in., respectively; what is the ratio of their perimeters?

2. The bases of two similar triangles are 2m. 5dm., and 7m. 5dm., respectively, and the perimeter of the first triangle is 9m; what is the perimeter of the second?

3. The base of a triangle is 10m. and its altitude is 15m; what is the length of a line drawn within the triangle parallel to the base and 9m. below the vertex?

4. The base of a triangle is 12 ft. and its altitude is 8 ft; what is the area of a trapezoid cut off by a line parallel to the base and 6 ft. from the vertex?

5. The base of a triangle is 14 in. and its altitude is 7 in; find the area of a trapezoid cut off by a line 6 in. from the vertex.

6. The perimeter of a certain triangle is 3 times the perimeter of a second similar triangle and the base of the first triangle is 9m. 6dm.; what is the base of the second triangle?

7. The perimeter of a certain triangle is 9m. 6dm. and its base is 2m. 4dm.; what is the base of a similar triangle whose perimeter is $\frac{2}{3}$ as great?

8. The altitude of a certain triangle is 8cm. 5mm.; what is the altitude of a similar triangle whose perimeter is $\frac{4}{5}$ that of the given triangle?

230. Draw a triangle ABC with its sides AB, BC, and CA equal, respectively, to $3\frac{1}{2}$ in., $3\frac{1}{2}$ in., and $3\frac{1}{2}$ in.

Draw another triangle A'B'C' with its sides A'B', B'C', and C'A' equal, respectively, to $2\frac{5}{8}$ in., $2\frac{7}{8}$ in., and $2\frac{1}{8}$ in.

Draw the corresponding altitudes CD and C'D'.

Measure these altitudes and mark their lengths on the figures.

What is the area of the triangle ABC? (See §192.)

What is the area of the triangle A'B'C'?

What is the ratio of the area of ABC to the area of $A'B'C'$?

What is the ratio of the corresponding sides AB and $A'B'$?

What is the ratio of the squares of AB and $A'B'$?

Is the ratio of the areas equal to the ratio of the squares of AB and $A'B'$?

Is the ratio of the areas also equal to the ratio of the squares of BC and $B'C'$? Of CA and $C'A'$?

What may be inferred from this in regard to *the ratio of the areas of any two similar triangles?*

Statement:—

231.

EXERCISES.

1. The area of a certain triangle is 48 sq. ft. and its base is 8 ft. The base of a similar triangle is 4 ft.; what is its area?

2. The area of a certain triangle is 100 sq. m. and its base is 6m. What is the base of a similar triangle whose area is 400 sq. m.?

3. The ratio of the areas of two similar triangles is 4:9; what is the ratio of any two corresponding sides?

4. The ratio of two corresponding sides of two similar triangles is 3:5; what is the ratio of their areas?

5. What is the ratio of the areas of two similar triangles whose bases are 3m. 6dm. and 7m. 2dm., respectively?

6. What is the ratio of any two corresponding sides of two similar triangles whose areas are 1 sq. m. 44 sq. dm. and 3 sq. m. 24 sq. dm., respectively?

7. What is the ratio of the altitudes of two similar triangles whose areas are 81 sq. ft. and 225 sq. ft.?

8. The perimeters of two similar triangles are 9dkm. 8m. and 2hm. 1dkm. 7m., respectively; what is the ratio of their areas?

232. Draw a circumference with centre O and radius 2cm. long.

Inscribe a regular hexagon in this circle and mark its vertices A, B, C , etc.

Draw a second circumference with centre O' and radius 3cm. long.

Inscribe a regular hexagon in this circle and mark its vertices $A'B'C'$, etc.

How many degrees are there in each angle of a regular hexagon? (See §165.)

Are these two hexagons mutually equiangular?

What is the ratio of AB to $A'B'$? Of BC to $B'C'$? Of CD to $C'D'$? etc.

Are the corresponding sides of the two hexagons proportional?

Are these two regular hexagons similar?

Give the reason for your answer.

What may be inferred from this in regard to *two regular polygons of the same number of sides*?

Statement:—

233. What are the lengths of the perimeters of the two similar polygons in Fig. 27, page 125?

What is the ratio of the perimeters?

What is the ratio of any two corresponding sides?

Is the ratio of the perimeters equal to the ratio of any two corresponding sides?

What may be inferred from this in regard to *the ratio of the perimeters of any two similar polygons?*

Statement:—

234.**EXERCISES.**

1. In two similar polygons, two corresponding sides are 27 ft. and 36 ft.; what is the perimeter of the second polygon, if the perimeter of the first is 240 ft.?

2. The corresponding sides of two similar polygons are in the ratio 4 : 5. The perimeter of the first polygon is 3m. 2dm. 4cm.; what is the perimeter of the second?

3. Of two regular hexagons, a side of the first is 5 in. and of the second 15 in.; what is the ratio of their perimeters?

4. Of two regular octagons, the perimeter of the first is 32m. and a side of the second is 5m.; what is the ratio of their sides?

5. Draw two regular hexagons whose sides will have the ratio 3 : 4.

6. The perimeter of a certain square is 3 times the length of one side of another square; what is the ratio of their perimeters?

7. Of two regular hexagons, a side of the first is $\frac{1}{3}$ of the perimeter of the second; what is the ratio of their perimeters?

235. Draw two squares, one having its sides 2cm. long and the other having its sides 3cm. long.

Divide each of these squares into square centimeters.

What is the ratio of the area of the smaller square to the area of the larger?

What is the ratio of a side of the smaller to the side of the larger?

What relation do you observe between these two ratios?

236. The two similar triangles in Fig. 28 are divided into small equal triangles.

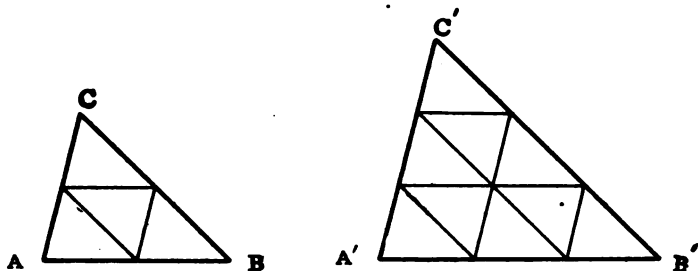


Fig. 28.

Count the number of small triangles in ABC and $A'B'C'$ and write the ratio of the area of ABC to the area of $A'B'C'$.

What is the ratio of AC to $A'C'$?

What relation do you observe between the ratio of the areas and the ratio of the corresponding sides?

Does this agree with the statement of §230?

237. Calculate the areas, in square inches, of the two similar polygons in Fig. 29 and write their ratio.

What is the ratio of the corresponding sides AB and $A'B'$? Of BC and $B'C'$? Of CD and $C'D'$? etc., etc.

Is the ratio of the areas equal to the ratio of the squares of any two corresponding sides?

What may be inferred from this in regard to the ratio of the areas of two similar polygons?

Statement:—

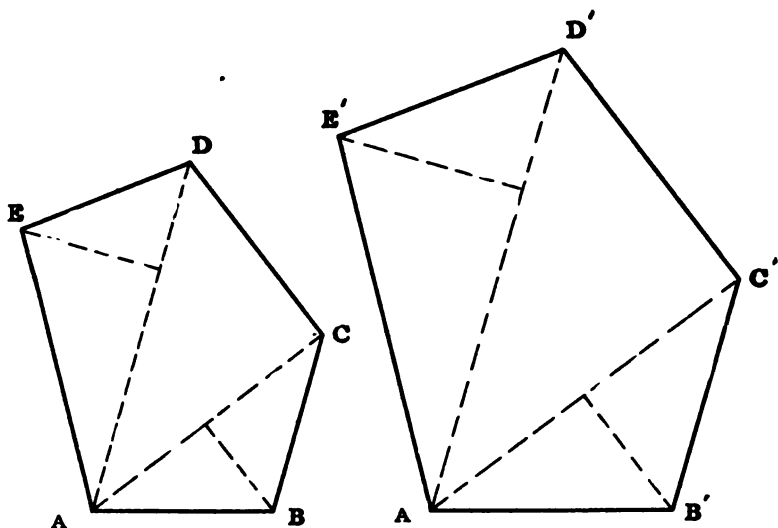


FIG. 29.

238.**EXERCISES.**

1. The ratio of the areas of two similar polygons is $16 : 25$; what is the ratio of any two corresponding sides?

2. The ratio of any two corresponding sides of two similar polygons is $3 : 5$; what is the ratio of their areas?

3. The bases of two similar polygons are 5 ft. and 7 ft., respectively, and the area of the first is 125 sq. ft.; what is the area of the second?

4. The areas of two similar polygons are 81 sq. m. and 4 sq. dkm. 41 sq. m., respectively, and the base of the second is 7m.; what is the base of the first?

5. The areas of two similar polygons are 96 sq. m. and 2 sq. dkm. 16 sq. m., respectively; what is the ratio of any two corresponding sides?

6. The base of a given polygon is 5 in. ; what is the base of a similar polygon whose area is nine times as great?

7. The area of a given polygon is 405 sq. yds. ; what is the area of a similar polygon whose base is $\frac{2}{3}$ that of the given polygon?

8. The bases of two similar polygons are 5 yds. and 6 yds., respectively ; what is the ratio of their areas?

9. Draw a rectangle having its base 3cm. long and its altitude 2cm.

Draw a second rectangle, similar to the first, having four times as much area.

10. Draw a square having its sides 4cm. long.

Draw a second square whose area is $\frac{1}{16}$ that of the first square.

11. The area of a given rectangle is 24 sq. ft. and its altitude is 3 ft. ; what is the base of a similar rectangle whose area is 25 times that of the given rectangle?

239. The radii of two circles are r and R ; what are their circumferences? (See §157.)

What is the ratio of their circumferences?

Have the circumferences of two circles the same ratio as their radii?

Write the answer below in the form of a complete statement.

Statement:—

240. The radii of two circles are r and R ; what are their areas? (See §202.)

What is the ratio of their areas?

Is the ratio of the areas equal to the ratio of the squares of the radii?

Write the answer below in the form of a complete statement.

Statement: —

241.

EXERCISES.

1. Draw a circle with a radius of 2cm. 5mm.

Draw a second circle whose circumference will be twice that of the given circle.

2. Draw a circle with a radius of 2cm.

Draw a second circle whose area will be four times that of the first.

3. The ratio of the radii of two circles is 3 : 5; what is the ratio of their areas?

4. The diameter of a given circle is 3m. 9dm.; what is the diameter of a second circle whose area is nine times that of the given circle?

5. The area of a given circle is 385 sq. ft.; what is the diameter of a second circle whose area is $\frac{5}{8}$ of the area of the given circle?

6. Draw a circle whose area will be equal to the sum of the areas of two circles whose radii are 1 cm. 5mm. and 2cm., respectively.

7. Draw a circle whose area will be equal to the difference of the areas of two circles whose radii are $1\frac{1}{4}$ in. and $\frac{3}{4}$ in., respectively.

EXAMINATION PAPERS.

I.

1. Define a right angle.
2. Define an angle of one degree.
3. Through how many degrees does the minute hand of a clock move in 13 minutes ?
4. Mention the exact hours when the hands of a clock form an acute angle.
5. Write a definition of adjacent angles.
6. One of two complementary angles is five times the other ; how many degrees are there in each ?
7. Define supplementary adjacent angles.
8. Of two angles having their sides parallel each to each, one is eight times the other ; how many degrees are there in each ?
Explain how you obtained your answer.
9. Draw two parallel lines cut by a transversal and letter the figure.
Name the alternate interior angles. The corresponding angles.
10. Draw any straight line and divide it into four equal parts by two different methods.

II.

1. Define a triangle.
2. Name three kinds of triangles and define each.
3. One angle of a certain triangle contains 15° , and, of the other two, one is twice as large as the other ; how many degrees are there in each of the other two ?
Explain how you obtained your result.

4. The three angles of a triangle, A, B, and C contain 40° , 75° , and 65° , respectively; how many degrees are there in the exterior angle at the vertex A? At the vertex B?

Explain how you obtained your result.

5. How many obtuse angles may a triangle have? Give the reason for your answer.

6. Construct a triangle having two of its sides 4cm. and 5cm. long, respectively, and the included angle equal to 70° .

7. What do you know in regard to the perpendicular bisectors of the sides of a triangle?

8. Define a median of a triangle?

9. How does the position of the common point of intersection of the three altitudes differ for an acute angle triangle, a right triangle, and an obtuse angle triangle?

10. How many degrees are there in each angle of an equilateral triangle?

III.

1. Name all the different kinds of quadrilaterals and define each.

2. What do you know in regard to the opposite sides of a parallelogram?

3. The angle A of a parallelogram ABCD contains 55° ; how many degrees are there in each of the other three?

4. The median of a certain trapezoid is 5 in. long, and the lower base is 2 in. longer than the upper base; how long is the upper base?

5. The area of one of the triangles formed by drawing a diagonal in a certain parallelogram is 55 sq. m. 35 sq. dm.; what is the area of the parallelogram?

Explain how you obtained your answer.

6. How many obtuse angles may a quadrilateral have?

7. What is the perimeter of the square formed by placing 9 small squares together, if the perimeter of each of the small squares is 12 inches?

IV.

1. Define an inscribed angle.
2. If the circumference of a circle is 225 m. long, how many degrees are there in the angle at the centre which intercepts an arc of 15 m.?
3. One pair of vertical angles, formed by two intersecting chords, intercepts two arcs which are, respectively, $\frac{1}{3}$ and $\frac{1}{4}$ of the circumference; how many degrees are there in each of these angles?
4. Define a tangent to a circle.
5. Draw a circle with a radius equal to 3 cm. and draw a tangent to this circle from an external point 5 cm. from the centre.
6. How many degrees are there in the angle formed by two secants that intercept arcs which are $\frac{1}{6}$ and $\frac{1}{3}$ of the circumference, respectively?
7. Draw any triangle and circumscribe a circle about it.
8. What is the length of the circumference of a circle whose radius is 49 in.?
9. Draw an equilateral triangle having each of its sides 2 cm. 8 mm. long.
With the vertices of the triangle as centres and a radius equal to 1 cm. 4 mm., draw three arcs outside the triangle and terminated by the sides of the triangle.
What is the total perimeter of the figure enclosed by these three arcs.
10. If the diameter of a circle is 5 in., what is the altitude of the equilateral triangle inscribed in it?

V.

1. Define a polygon.
2. What is the sum of the interior angles of a polygon of n sides?

3. What name would you give to a polygon, if the sum of its interior angles is equal to eight right angles?

4. Define a regular polygon.

5. Inscribe a regular hexagon in a circle of which the radius is 1 cm. 9 mm.

What is the perimeter of this hexagon?

What is the length of its longest diagonal?

6. How many degrees are there in each angle at the centre of a regular octagon?

7. How many sides has a regular polygon, if the angles at the centre each contain 15° ?

8. Inscribe a regular pentagon in a circle.

VI.

1. Construct a rectangle having its base and altitude equal, respectively, to 5 cm. and 3 cm. and explain how the product of the base and altitude gives the numerical measure of its area.

2. If the length of the hypotenuse of a right triangle is one and one quarter times the longer leg and the area of the square constructed on the shorter leg is 36 sq. ft., what is the length of each of the three sides of the triangle?

3. The sum of the diagonals of a rhombus is 24 yds. and one of them is twice as long as the other; what is the area of the rhombus?

4. To what is the area of a trapezoid equal?

5. Inscribe a regular hexagon in a circle and find the area of the hexagon. Measure the required lengths with the scale.

6. Write the expression for the area of a circle in terms of its radius.

7. What is the area of a sector whose angle is 54° and whose radius is 56 in.?

8. The circumference of a circle and the perimeter of a

square are each equal to 484 meters; which has the greater area, and what is the difference in their areas?

VII.

1. Draw two straight lines, one 3 cm. and the other 5 cm. in length, and find a third proportional to them.

2. Draw a right triangle and drop a perpendicular from the vertex of the right angle to the hypotenuse.

Letter the figure and tell all the facts you know in regard to it.

3. Define similar polygons.

4. The sides of a certain triangle are 5, 6, and 7; in a similar triangle the side corresponding to 6 is 24; what are the other two sides of the second triangle?

5. What is the relation between the ratio of the areas of two similar polygons and the ratio of any two homologous sides?

6. If the area of a certain polygon is 270 sq. m., what is the area of a similar polygon whose base is $\frac{2}{3}$ of the base of the given polygon?

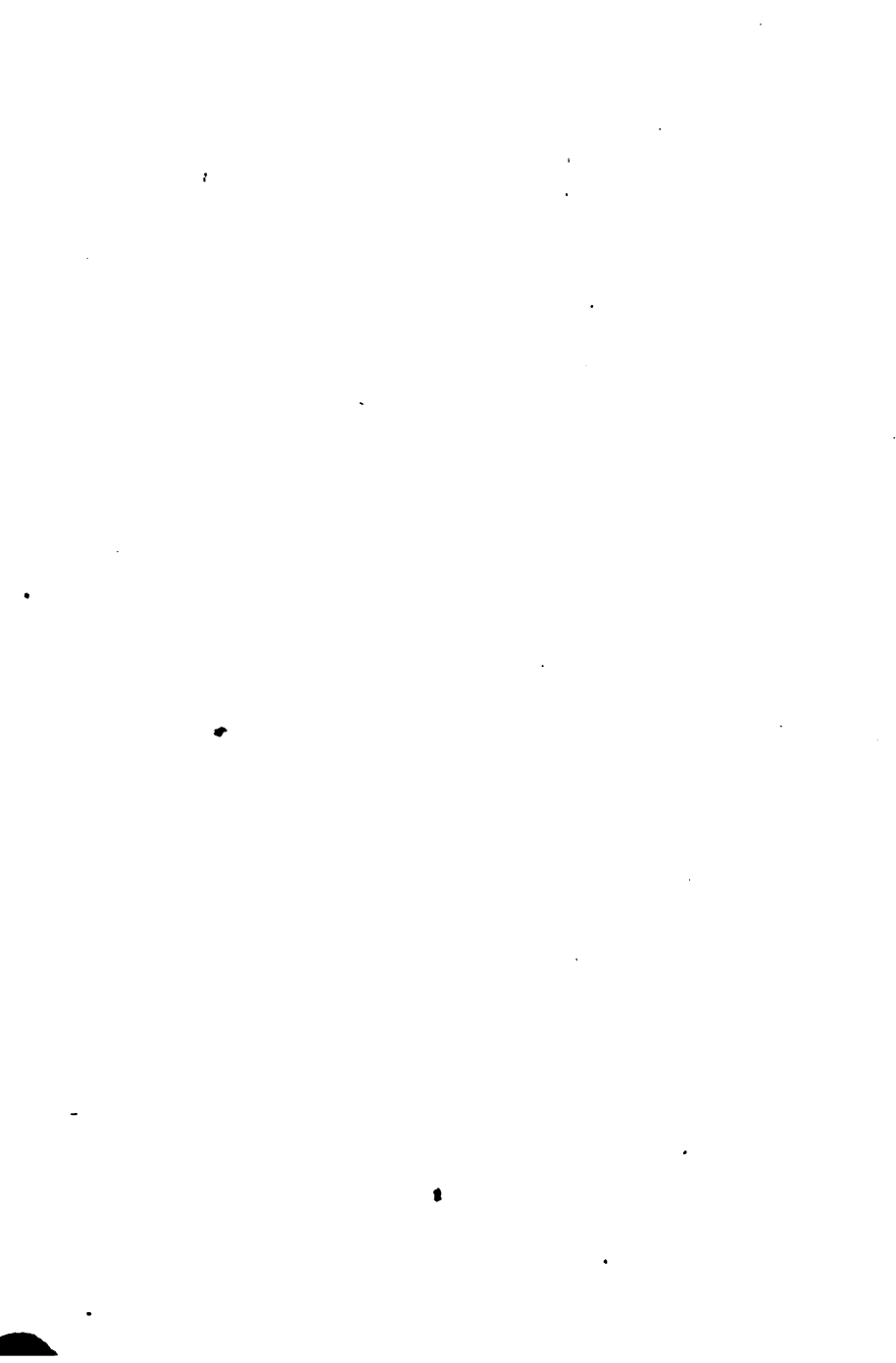
7. What is the relation between two regular polygons of the same number of sides?

8. The area of a given circle is 770 sq. ft.; what is the length of the diameter of a circle whose area is $\frac{1}{4}$ that of the given circle?

9. What do you know in regard to the division of two sides of a triangle by a line parallel to the base?

10. What do you know in regard to the bisector of an exterior angle of a triangle?





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